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SIMPLIFIED EARTHQUAKE ANALYSIS OF GATED SPILLWAY MONOLITHS OF CONCRETE GRAVITY DAMS

by

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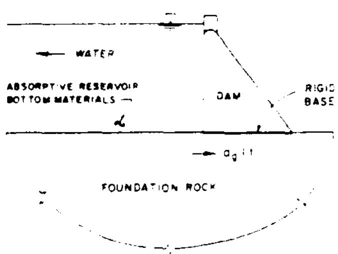

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The simplified procedure for earthquake analysis of concrete gravity dams developed earlier for nonoverflow monoliths is extended in this report to gated spillway monoliths. Standard data are presented for the vibration properties of such monoliths and for all parameters that are required in the analysis procedure. The use of the simplified analysis procedure and of a computer program that facilitates implementation of the procedure is illustrated by examples.

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PREFACE

This report presents a simplified procedure for earthquake analysis of concrete gravity dams for gated spillway monoliths, an extension of an earlier report on nonoverflow monoliths. Standard data are presented for the vibration properties of such monoliths and for all parameters that are required in the analysis procedure. The use of the simplified analysis procedure and of a computer program that facilitates implementation of the procedure is illustrated by examples.

This report was prepared by Dr. Anil K. Chopra, Professor of Civil Engineering, and Hanchen Tan, graduate student, University of California at Berkeley. Funds for this report were provided to the US Army Engineer Waterways Experiment Station (WES) by the Civil Works Research and Development Program of the Office, Chief of Engineers (OCE), under the Structural Engineering Research Program, Contract No. DAAG29-81-D-0100, Delivery Order No. 1855 with Battelle Laboratories, Research Triangle Park, NC.

Vincent P. Chiarito and Dr. Robert L. Hall of the Structural Mechanics Division (SMD), Structures Laboratory (SL), WES, and Lucien G. Guthrie, OCE, Engineering Division, provided input for this work. Dr. Hall prepared the preliminary computer analyses (Reference 6) requested by the authors in order to develop the system idealization used in this report for gated spillway monoliths.

Although this report could have been written as an addendum to the work on nonoverflow monoliths, for the convenience of the user it is organized to be self-contained, but at the expense of extensive duplication with References 2 and 3, which resulted from the work of Dr. Gregory L. Fenves. The investigation was conducted at WES under the general supervision of Messrs. Bryant Mather, Chief, SL, James T. Ballard, Assistant Chief, SL, and Dr. Jimmy P. Balsara, Chief, SMD, SL.

COL Dwayne G. Lee, EN, is the Commander and Director of WES. Dr. Robert W. Whalin is Technical Director.

CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
inches	25.4	millimetres
kips (force)	4.448222	kilonewtons
kips (force) per square inch	6.894757	megapascals
miles (US statue)	1.609	kilometres
pounds (force)	4.448222	newtons
pounds (force) per foot	14.593904	newtons per metre
pounds (force) per square inch	0.006894757	megapascals

SIMPLIFIED EARTHQUAKE ANALYSIS OF GATED SPILLWAY MONOLITHS OF CONCRETE GRAVITY DAMS

INTRODUCTION

A simplified analysis procedure was presented in 1978 for elastic design and safety evaluation of concrete dams [1]. In this procedure the lateral earthquake forces associated with the fundamental vibration mode of nonoverflow monoliths are estimated directly from the earthquake design spectrum, considering the effects of dam-water interaction and water compressibility. More recently [2,3], the procedure has been extended to also consider the absorption of hydrodynamic pressure waves in the reservoir bottom sediments and in the underlying foundation rock. Also included in the newer simplified procedure is a "static correction" method to consider the response contributions of the higher vibration modes, and a rule for combining the modal responses.

The objective of this report is to extend the above-mentioned simplified analysis procedure for nonoverflow monoliths to gated spillway monoliths. Utilizing the analytical development underlying the procedure [4,5], this report is concerned with implementation of the procedure. Standard data for the vibration properties of gated spillway monoliths and the quantities that depend on them are presented to minimize the computations. The use of the simplified analysis procedure and a computer program for static stress analysis is illustrated by examples.

SYSTEM IDEALIZATION

A gated spillway monolith of a concrete gravity dam is shown in Fig. 1, including a pier, gate, bridge, and foot bucket. Based on results of finite element analyses, it was concluded that: (1) the fundamental vibration period and mode shape of a spillway monolith are not influenced significantly by the bridge, gate, or foot bucket [6], but the effects of the pier may not be negligible; and (2) an equivalent two-dimensional system of unit thickness along the dam axis, with the mass and elastic modulus of the monolith kept at their actual values, but those of the pier reduced by the ratio of monolith thickness to pier thickness, is satisfactory for computing the fundamental vibration period and mode shape of the system [6].

The equivalent two-dimensional system representing a gated spillway monolith is assumed to be supported on a viscoelastic half-plane and impounding a reservoir of water, possibly with alluvium and sediments at the bottom (Fig. 2). Although the equivalent single-degree-of-freedom (SDF) system representation is valid for dams of any cross-section, the upstream face of the dam was assumed to be vertical [4,5] only for the purpose of evaluating the hydrodynamic terms in the governing equations. The standard data presented in this report are also based on this assumption, which is reasonable for actual concrete gravity dams because the upstream face is vertical or almost vertical for most of the height, and the hydrodynamic pressure on the dam face is insensitive to small departures of the face slope from vertical, especially if these departures are near the base of the dam, which is usually the case. The dynamic effects of the tail water are neglected because it is usually too shallow to influence dam response.

EQUIVALENT LATERAL FORCES FOR FUNDAMENTAL VIBRATION MODE

Considering only the fundamental mode of vibration of the dam, the maximum effects of the horizontal earthquake ground motion can be represented by equivalent lateral forces acting on the upstream face of the spillway monolith and pier [2,3,5]:

$$f_1(y) = \frac{\tilde{L}_1}{\tilde{M}_1} \frac{S_a(\tilde{T}_1, \tilde{\xi}_1)}{g} [w_s(y)\,\phi(y) + gp(y, \tilde{T}_r)] \tag{1}$$

in which the y-coordinate is measured from the base of the dam along its height and $w_s(y)$ is the weight per unit height of the equivalent two-dimensional system of unit thickness along the dam axis. As mentioned in the preceding section, $w_s(y)$ is equal to the actual weight of the dam monolith per unit thickness, but for the pier it is the weight of the pier per unit thickness divided by the ratio of the monolith thickness to the pier thickness. In Eq. 1,

$$\tilde{M}_1 = M_1 + \int_0^H p(y, \tilde{T}_r)\,\phi(y)\,dy \tag{2a}$$

$$M_1 = \frac{1}{g}\int_0^{H_s} w_s(y)\,\phi^2(y)\,dy \tag{2b}$$

$$\tilde{L}_1 = L_1 + \int_0^H p(y, \tilde{T}_r)\,dy \tag{3a}$$

$$L_1 = \frac{1}{g}\int_0^{H_s} w_s(y)\,\phi(y)\,dy \tag{3b}$$

M_1 is the generalized mass and L_1 is the generalized earthquake force coefficient; $\phi(y)$ is the horizontal component of displacement at the upstream face of the dam in the fundamental vibration mode shape of the dam supported on rigid foundation rock with empty reservoir; $p(y, T_r) \equiv Re[\overline{p}_1(y, \tilde{T}_r)]$ where $\overline{p}_1$ is the complex-valued function representing the hydrodynamic pressure on the upstream face due to harmonic acceleration of period $\tilde{T}_r$ (defined later) in the fundamental vibration mode; H is the depth of the impounded water; H_s is the height of the dam; g is the acceleration due to gravity; and $S_a(\tilde{T}_1, \xi_1)$ is the pseudo-acceleration ordinate of the earthquake design spectrum evaluated at the vibration period $\tilde{T}_1$ and damping ratio ξ_1 of the equivalent SDF system representing the dam-water-foundation rock system. Equation 1 is an extension of Eq. 9 in Ref. 1 to include the effects of dam-foundation rock interaction and reservoir bottom materials on the lateral forces.

The natural vibration period of the equivalent SDF system representing the fundamental mode response of the dam on rigid foundation rock with impounded water is [4]:

$$\tilde{T}_r = R_r T_1 \tag{4a}$$

in which T_1 is the fundamental vibration period of the dam on rigid foundation rock with empty reservoir. Because of the frequency-dependent, added hydrodynamic mass arising from dam-water interaction, the factor $R_r > 1$. It depends on the properties of the dam, the depth of the water, and the absorptiveness of the reservoir bottom materials. The natural vibration period of the equivalent SDF system representing the fundamental mode response of the dam on flexible foundation rock with emptry reservoir is [4]:

$$\tilde{T}_f = R_f T_1 \tag{4b}$$

Because of the frequency-dependent, added foundation-rock flexibility arising from dam-foundation rock interaction, the factor $R_f > 1$. It depends on the properties of the dam and foundation rock.

The natural vibration period of the equivalent SDF system representing the fundamental mode response of the dam on flexible foundation rock with impounded water is approximately given by [5]:

$$\tilde{T}_1 = R_r R_f T_1 \tag{4c}$$

The damping ratio of this equivalent SDF system is [5]:

$$\tilde{\xi}_1 = \frac{1}{R_r} \frac{1}{(R_f)^3} \xi_1 + \xi_r + \xi_f \tag{5}$$

in which ξ_1 is the damping ratio of the dam on rigid foundation rock with emptry reservoir; ξ_r represents the added damping due to dam-water interaction and reservoir bottom absorption; and ξ_f represents the added radiation and material damping due to dam-foundation rock interaction. Considering that R_r and $R_f > 1$, Eq. 5 shows that dam-water interaction and dam-foundation rock interaction reduce the effectiveness of structural damping. However, usually, this reduction is more than compensated for by the added damping due to reservoir bottom absorption and due to dam-foundation rock interaction, which leads to an increase in the overall damping of the dam.

The quantities R_r, R_f, ξ_r, ξ_f, $p(y, \tilde{T}_r)$, $\tilde{L}_1$, and $\tilde{M}_1$ which are required to evaluate the equivalent lateral forces, Eq. 1, contain all the modifications of the vibration properties of the equivalent SDF system and of the generalized earthquake force coefficient necessary to account for the effecs of dam-water interaction, reservoir bottom absorption, and dam-foundation rock interaction. Even after the considerable simplification necessary to arrive at Eq. 1, its evaluation is still too complicated for practical applications because the aforementioned quantities are complicated fucntions of the hydrodynamic and foundation-rock flexibility terms [5]. Fortunately, as will be seen in a later section, the computation of lateral forces can be considerably simplified by recognizing that the cross-sectional geometry of concrete gravity dams does not vary widely.

STANDARD PROPERTIES FOR FUNDAMENTAL MODE RESPONSE

Direct evaluation of Eq. 1 would require complicated computation of several quantities: $p(y, \tilde{T}_r)$ from an infinite series expression; the period lengthening ratios R_r and R_f due to dam-water and dam-foundation rock interactions by iterative solution of equations involving frequency-dependent terms; damping ratios ξ_f and ξ_r from expressions involving complicated foundation-rock flexibility and hydrodynamic terms; the integrals in Eqs. 2a and 3a; and the fundamental vibration period and mode shape of the dam [4,5]. The required computations would be excessive for purposes of preliminary design of dams. Recognizing that the cross-section geometry of nonoverflow monoliths of concrete gravity dams does not vary widely, standard values for the vibration properties of these monoliths and all quantities that depend on them and enter into Eq. 1 were developed in Refs. 2 and 3. A single set of standard data was sufficient because the standard shape chosen for nonoverflow monoliths was assumed to be appropriate for dams of all heights; i.e. the standard shape multiplied by a height-scaling factor provides the cross-section for a dam of particular height.

However, a single standard shape did not seem appropriate for spillway monoliths because, according to Corps of Engineers staff, the pier height is usually about the same (approximately 60 ft) for a wide range of dam heights. Because a pier of these dimensions would influence the vibration properties of shorter monoliths to a greater degree than it would affect taller monoliths, it was decided to generate two sets of "standard" data, one appropriate for lower dams, defined for purposes of this report as $H_s < 300$ ft; the other for higher dams, defined herein as $H_s \geq 300$ ft. Because the slopes of the downstream and upstream faces of the monolith are usually steeper in higher dams compared to lower dams, cross-sectional geometry appropriate to each of the two cases was selected (Appendix A) in generating the data presented later.

Vibration Properties of the Dam

Computed by the finite element method, the fundamental vibration period, in seconds, of two "standard" cross-sections for spillway monoliths of concrete gravity dams on rigid foundation rock with empty reservoir is (Appendix A):

$$T_1 = 1.2 \frac{H_s}{\sqrt{E_s}} \quad \text{if } H_s \geq 300 \text{ ft} \tag{6a}$$

$$T_1 = 1.25 \frac{H_s}{\sqrt{E_s}} \quad \text{if } H_s < 300 \text{ ft} \tag{6b}$$

in which H_s is the total (monolith plus pier) height of the dam, in feet; and E_s is the Young's modulus of elasticity of concrete, in pounds per square inch. The fundamental vibration mode shape $\phi(y)$ of the two standard cross-sections is shown in Fig. 3 and Table 1.

Modification of Period and Damping: Dam-Water Interaction

Dam-water interaction and reservoir bottom absorption modify the natural vibration period (Eq. 4a) and the damping ratio (Eq. 5) of the equivalent SDF system representing the fundamental vibration mode response of the dam. For a fixed cross-section, the period lengthening ratio R_r and added damping ξ_r depend on several parameters, the more significant of which are: Young's modulus E_s of the dam concrete, ratio H/H_s of water depth to dam height, and wave reflection coefficient α. This coefficient, α, is the ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of a vertically propagating pressure wave incident on the reservoir bottom [4,7,8,9]; $\alpha = 1$ indicates that pressure waves are completed reflected, and smaller values of α indicate increasingly absorptive materials.

The results of many analyses of the two "standard" spillway cross-sections, using the procedures developed in Ref. 4 and modified in Ref. 2 for dams with larger elastic modulus E_s, are summarized in Figs. 4 and 5 and Table 2 for higher dams ($H_s \geq 300$ ft), and in Figs. 6 and 7 and Table 3 for lower dams ($H_s < 300$ ft). The period lengthening ratio R_r and added damping ξ_r are presented as a function of H/H_s for $E_s = 5.0$, 4.5, 4.0, 3.5, 3.0, 2.5, and 2.0 million psi and $\alpha = 1.00$, 0.90, 0.75, 0.50, 0.25, and 0. Whereas the dependence of R_r and ξ_r on E_s, H/H_s and α, and the underlying mechanics of dam-water interaction and reservoir bottom absorption are discussed elsewhere in detail [2,4,5], it is useful to note that R_r increases and ξ_r generally, but not always, increases with increasing water depth, absorptiveness of reservoir bottom materials, and concrete modulus. The effects of dam-water interaction and reservoir bottom absorption may be neglected, and the dam analyzed as if there is no impounded water, if the reservoir depth is small, $H/H_s < 0.5$; in particular, $R_r \approx 1$ and $\xi_r \approx 0$.

Modification of Period and Damping: Dam-Foundation Rock Interaction

Dam-foundation rock interaction modifies the natural vibration period (Eq. 4b) and added damping ratio (Eq. 5) of the equivalent SDF system representing the fundamental vibration mode response of the dam. For a fixed dam cross-section, the period lengthening ratio R_f and the added damping ξ_f due to dam-foundation rock interaction depend on several parameters, the more significant of which are: moduli ratio E_f/E_s, where E_s and E_f are the Young's moduli of the dam concrete and foundation rock, respectively; and the constant hysteretic damping factor η_f for the foundation rock.

The results of many analyses of the two "standard" dam cross-sections, using the procedures developed in Ref. 4, are summarized in Figs. 8-9 and Table 4 for higher dams ($H_s \geq 300$ ft) and in Figs. 10-11 and Table 5 for lower dams ($H_s < 300$ ft). The period lengthening ratio R_f and added damping ξ_f are presented for many values of E_f/E_s between 0.2 and 5.0, and $\eta_f = 0.01$, 0.10, 0.25, and 0.50. Whereas the dependence of R_f and ξ_f on E_f/E_s and η_f, and the underlying mechanics of dam-foundation rock interaction are discussed elsewhere in detail [4,5], it is useful to note that the period ratio R_f and added damping ξ_f increase with decreasing E_f/E_s--which, for a fixed value of E_s, implies an increasingly flexible foundation rock--and increasing hysteretic damping factor η_f. The foundation rock may be considered rigid in the simplified analysis if $E_f/E_s > 4$ because then the effects of dam-foundation rock interaction are negligible.

Hydrodynamic Pressure

In order to provide a convenient means for determining $p(y, \tilde{T}_r)$ in Eqs. 1, 2a and 3a, a nondimensional form of this function, $gp(\hat{y})/wH$, where $\hat{y} = y/H$, and w = the unit weight of water, has been computed from the equations presented in Ref. 4 for $\alpha = 1.0$, 0.90, 0.75, 0.5, 0.25, and 0, and the necessary range of values of

$$R_w = \frac{T_1^r}{\tilde{T}_r} \tag{7}$$

in which the fundamental vibration period of the impounded water $T_1^r = 4H/C$, where C is the velocity of pressure waves in water. The results presented in Fig. 12 and Table 6 are for full reservoir, $H/H_s = 1$. The function $gp(\hat{y})/wH$ for any other value of H/H_s is approximately equal to $(H/H_s)^2$ times the function for $H/H_s = 1$ [1].

Generalized Mass and Earthquake Force Coefficient

The generalized mass $\tilde{M}_1$ (Eq. 2a) of the equivalent SDF system representing the dam, including hydrodynamic effects, can be conveniently cmputed from [4]:

$$\tilde{M}_1 = (R_r)^2 M_1 \tag{8a}$$

in which M_1 is given by Eq. 2b. In order to provide a convenient means to compute the generalized earthquake force coefficient $\tilde{L}_1$, Eq. 3a is expressed as:

$$\tilde{L}_1 = L_1 + \frac{1}{g} F_{st} \left(\frac{H}{H_s} \right)^2 A_p \tag{8b}$$

where $F_{st} = wH^2/2$ is the total hydrodynamic force on the dam, and A_p is the integral of the function $2gp(\hat{y})/wH$ over the depth of the impounded water, for $H/H_s = 1$. The hydrodynamic force coefficient A_p is tabulated in Table 7 for a range of values for the period ratio R_w and the wave reflection coefficient α.

EQUIVALENT LATERAL FORCES FOR HIGHER VIBRATION MODES

Because the earthquake response of short vibration period structures, such as concrete gravity dams, is primarily due to the fundamental mode of vibration, the response contributions of the higher vibration modes have, so far, been neglected in the simplified analysis procedure presented in the preceding sections. However, the height-wise mass distribution of concrete gravity dams is such that the effective mass [10] in the fundamental vibration mode is small, e.g. it is 35 percent of the total mass for the standard nonoverflow section [2]. Thus, the contributions of the higher vibration modes to the earthquake forces may not be negligible, and a simple method to consider them is summarized in this section.

This simple method utilizes three concepts. Firstly, because the periods of the higher vibration modes of concrete gravity dams are very short, the higher vibration modes respond to earthquake ground motion with little dynamic amplification in essentially a static manner, leading to the "static correction" concept [11,12]. Secondly, just as in the case of multistory buildings [13], soil-structure interaction effects may be neglected in computing the contributions of the higher vibration modes to the earthquake response of dams. Thirdly, the effects of dam-water interaction and water compressibility may be neglected in computing the higher mode responses [2]. The maximum earthquake effects associated with the higher vibration modes can then be represented by the equivalent lateral forces [2]:

$$f_{sc}(y) = \frac{a_g}{g}\left\{ w_s(y)\left[1 - \frac{L_1}{M_1}\phi(y)\right] + \left[g\,p_0(y) - \frac{B_1}{M_1} w_s(y)\phi(y)\right]\right\} \tag{9}$$

In Eq. 9, a_g is the maximum ground acceleration, $p_0(y)$ is a real-valued, frequency-independent function for hydrodynamic pressure on a rigid dam undergoing unit acceleration, with water compressibility neglected, both assumptions being consistent with the "static correction" concept; and B_1 provides a measure of the portion of $p_0(y)$ that acts in the fundamental vibration mode. Standard values for $p_0(y)$ are presented graphically in Fig. 13 and Table 8. Using the fundamental mode vibration properties of the two "standard" spillway cross-sections, it can be shown that:

$$B_1 = 0.25\frac{F_{st}}{g}\left(\frac{H}{H_s}\right)^2 \tag{10}$$

for both cross-sections, where F_{st} is the total hydrostatic force on the dam. The shape of only the fundamental vibration mode enters into Eq. 9 and the higher mode shapes are not required, thus simplifying the analysis considerably.

RESPONSE COMBINATION

Dynamic Response

As shown in the preceding two sections, the maximum effects of earthquake ground motion in the fundamental vibration mode of the dam have been represented by equivalent lateral forces $f_1(y)$ and those due to all the higher modes by $f_{sc}(y)$. Static analysis of the dam for these two sets of forces provide the values r_1 and r_{sc} for any response quantity r, e.g. the shear force or bending moment at any horizontal section, or the shear or bending stresses at any point. Because the maximum responses r_1 and r_{sc} do not occur at the same time during the earthquake, they should be combined to obtain an estimate of the dynamic response r_d according to the well known modal combination rule: square-root-of-the-sum-of-squares (SRSS) of modal maxima leading to

$$r_d = \sqrt{(r_1)^2 + (r_{sc})^2} \tag{11}$$

Because the natural frequencies of lateral vibration of a concrete dam are well separated, it is not necessary to include the correlation of modal responses in Eq. 11. In Ref. 2, the SRSS combination rule is shown to be preferable to the sum-of-absolute-values (ABSUM) which may provide an overly conservative result.

The SRSS and ABSUM combination rules are applicable to the computation of any response quantity that is proportional to the generalized modal coordinate responses. Thus, these combination rules are generally inappropriate to determine the principal stresses. However, as shown in Ref. 2, the principal stresses at the faces of a dam monolith may be determined by the SRSS method if the upstream face is nearly vertical and the effects of tail water at the downstream face are small.

Total Response

In order to obtain the total value of any response quantity r, the SRSS estimate of dynamic response r_d should be combined with the static effects r_{st}. The latter may be determined by standard analysis procedures to compute the initial stresses in a dam prior to the earthquake, including effects of the self-weight of the dam, hydrostatic pressures, and temperature changes. In order to recognize that the direction of lateral earthquake forces is reversible, combinations of static and dynamic stresses should allow for the worst case, leading to the maximum value of total response:

$$r_{\max} = r_{st} \pm \sqrt{(r_1)^2 + (r_{sc})^2} \tag{12}$$

This combination of static and dynamic responses is appropriate if r_{st}, r_1, and r_{sc} are oriented similarly. Such is the case for the shearing force or bending moment at any horizontal section, for the shear and bending stresses at any point, but generally not for principal stresses, except under the restricted conditions mentioned above.

SIMPLIFIED ANALYSIS PROCEDURE

The maximum effects of an earthquake on a concrete gravity dam are represented by equivalent lateral forces in the simplified analysis procedure. The lateral forces associated with the fundamental vibration mode are computed to include the effects of dam-water interaction, water compressibility, reservoir bottom absorption, and dam-foundation rock interaction. The response contributions of the higher vibration modes are computed under the assumption that the dynamic amplification of the modes is negligible, the interaction effects between the dam, impounded water, and foundation rock are not significant, and that the effects of water compressibility can be neglected. These approximations provide a practical method for including the most important factors that affect the earthquake response of concrete gravity dams.

Selection of System Parameters

The simplified analysis procedure requires only a few parameters to describe the dam-water-foundation rock system: E_s, ξ_1, H_s, E_f, η_f, H, and α. The complete data necessary to implement this procedure are presented as both figures and tables in this report.

The Young's modulus of elasticity E_s for the dam concrete should be based on the design strength of the concrete or suitable test data, if available. The value of E_s may be modified to recognize the strain rates representative of those the concrete may experience during earthquake motions of the dam [1]. In using the figures and tables mentioned earlier to conservatively include dam-water interaction effects in the computation of earthquake forces (Eq. 1), the E_s value should be rounded down to the nearest value for which data are available: $E_s = 2.0$, 2.5, 3.0, 3.5, 4.0, 4.5 or 5.0 million psi. Forced vibration tests on dams indicate that the viscous damping ratio ξ_1 for concrete dams is in the range of 1 to 3 percent. However, for the large motions and high stresses expected in a dam during intense earthquakes, $\xi_1 = 5$ percent is recommended. The height H_s of the dam is measured from the base and includes the height of the spillway monolith and of the pier.

The Young's modulus of elasticity E_f and constant hysteretic damping coefficient η_f of the foundation rock should be determined from a site investigation and appropriate tests. To be conservative, the value of η_f should be rounded down to the nearest value for which data are available: $\eta_f = 0.01$, 0.10, 0.25, or 0.50, and the value of E_f/E_s should be rounded up to the nearest value for which data are available. In the absence of information on damping properties of the foundation rock, a value of $\eta_f = 0.10$ is recommended.

The depth H of the impounded water is measured from the free surface to the reservoir bottom. It is not necessary for the reservoir bottom and dam base to be at the same elevation. The standard values for unit height of water and velocity of pressures waves in water are $w =$ 62.4 pcf and $C = 4720$ fps, respectively.

It may be impractical to reliably determine the wave reflection coefficient α because the reservoir bottom materials may consist of highly variable layers of exposed bedrock, alluvium, silt, and other sediments, and appropriate site investigation techniques have not been developed. Until such techniques become available, α should be selected to give conservative estimates of the earthquake response, which is appropriate at the preliminary design stage. The wave reflection coefficient is defined as [4,7,8,9]:

$$\alpha = \frac{1 - qC}{1 + qC} \tag{13}$$

where $q = \rho/\rho_r C_r$, $C_r = \sqrt{E_r/\rho_r}$, E_r is the Young's modulus of elasticity of the reservoir bottom materials, and ρ_r is their density, C is the velocity of sound in water, and ρ is the density of water. For rigid reservoir bottom, $C_r = \infty$ and $q = 0$, resulting in $\alpha = 1$. In order to obtain a conservative value of α, the value of q may be based on the properties of the impounded water and only the underlying rock, thus neglecting the additional wave absorptiveness due to the overlying sediments. The estimated value of α should be rounded up to the nearest value for which the figures and tables are presented: $\alpha = 1.0, 0.90, 0.75, 0.50, 0.25$, and 0.00.

Design Earthquake Spectrum

The horizontal earthquake ground motion is specified by a pseudo-acceleration response spectrum in the simplified analysis procedure. This should be a smooth response spectrum--without the irregularities inherent in response spectra of individual ground motions--representative of the intensity and frequency characteristics of the design earthquakes which should be established after a thorough seismological and geological investigation (see Ref. 1 for more detail).

Computational Steps

The computation of the earthquake response of the dam (spillway monolith plus pier) is organized in three parts:

Part I: The earthquake forces and stresses due to the fundamental vibration mode can be determined approximately for purposes of preliminary design by the following computational steps:

1. Compute T_1, the fundamental vibration period of the dam, in seconds, on rigid foundation rock with an empty reservoir from Eq. 6 in which H_s = total (monolith plus pier) height of the dam in feet, and E_s = design value for Young's modulus of elasticity of concrete, in pounds per square inch.

2. Compute $\tilde{T}_r$, the fundamental vibration period of the dam, in seconds, including the influence of impounded water from Eq. 4a in which T_1 was computed in Step 1; R_r = period ratio determined from Figs. 4 and 5 or Tables 2 and 3 for the design values of E_s, the wave reflection coefficient α, and the depth ratio H/H_s, where H is the depth of the impounded water, in feet. If $H/H_s < 0.5$, computation of R_r may be avoided by using $R_r \approx 1$. Values for R_r are presented for higher dams ($H_s \geq 300$ ft)) in Fig. 4 and Table 2, and for lower dams ($H_s < 300$ ft) in Fig. 5 and Table 3.

3. Compute the period ratio R_w from Eq. 7 in which $\tilde{T}r$ was computed in Step 2 and $T_1^r = 4H/C$, where C = 4720 feet per second.

4. Compute $\tilde{T}_1$, the fundamental vibration period the dam in seconds, including the influence of foundation flexibility and of impounded water, from Eq. 4c in which R_r was determined from Step 2; R_f = period ratio determined for the design value of E_f/E_s from Fig. 8 or Table 4 for dams with $H_s \geq 300$ ft or from Fig. 10 or Table 5 if $H_s < 300$ ft. If $E_f/E_s > 4$, use $R_f \approx 1$.

5. Compute the damping ratio $\tilde{\xi}_1$ of the dam from Eq. 5 using the period ratios R_r and R_f determined in Steps 2 and 4, respectively; ξ_1 = viscous damping ratio for the dam on rigid foundation rock with empty reservoir; ξ_r = added damping ratio due to dam-water interaction and reservoir bottom absorption, obtained from Fig. 6 or Table 2 for dams with $H_s \geq 300$ ft, or from Fig. 7 or Table 3 if $H_s < 300$ ft, for the selected values of E_s, α, and H/H_s; and ξ_f = added damping ratio due to dam-foundation rock interaction, obtained from Fig. 9 or Table 4 for dams with $H_s \geq 300$ ft, or Fig. 11 or Table 5 if $H_s < 300$ ft, for the design values of E_f/E_s and η_f. If $H/H_s < 0.5$, use $\xi_r = 0$; if $E_f/E_s > 4$, use $\xi_f = 0$; and if the computed value of $\tilde{\xi}_1 < \xi_1$, use $\tilde{\xi}_1 = \xi_1$.

6. Determine $gp(y, \tilde{T}_r)$ from Fig. 12 or Table 6 corresponding to the value of R_w computed in Step 3--rounded to one of the two nearest available values, the one giving the larger $p(y)$ --the design value of α, and for $H/H_s = 1$; the result is multiplied by $(H/H_s)^2$. If $H/H_s < 0.5$, computation of $p(y, \tilde{T}_r)$ may be avoided using $p(y, \tilde{T}_r) \approx 0$..

7. Compute the generalized mass $\tilde{M}_1$ from Eq. 8a, in which R_r was computed in Step 2, and M_1 is computed from Eq. 2b, in which $w_s(y)$ = the weight per unit height of the equivalent two-dimensional system of unit thickness representing the dam (see page 2); the fundamen-

tal vibration mode shape $\phi(y)$ is given in Fig. 3 or Table 1; and g = 32.2 feet per squared second. Evaluation of Eq. 2b may be avoided by obtaining an approximate value from $M_1 = 0.060\, W_s/g$, where W_s is the total weight of the equivalent two-dimensional system.

8. Compute the effective earthquake force coefficient $\tilde{L}_1$ from Eq. 8b in which L_1 is computed from Eq. 3b; $F_{st} = wH^2/2$; and A_p is given in Table 7 for the values of R_w and α used in Step 6. If $H/H_s < 0.5$, computation of $\tilde{L}_1$ may be avoided by using $\tilde{L}_1 \approx L_1$. Evaluation of Eq. 3b may be avoided by obtaining an approximate value from $L_1 = 0.17\, W_s/g$.

   Note: Computation of Steps 7 and 8 may be avoided by using conservative values: $\tilde{L}_1/\tilde{M}_1 = 4$ for dams with impounded water, and $L_1/M_1 = 3$ for dams with empty reservoirs.

9. Compute $f_1(y)$, the equivalent lateral earthquake forces associated with the fundamental vibration mode from Eq. 1 in which $S_a(\tilde{T}_1, \tilde{\xi}_1)$ = the pseudo-accleration ordinate of the earthquake design spectrum in feet per square second at period $\tilde{T}_1$ determined in Step 4 and damping ratio $\tilde{\xi}_1$ determined in Step 5; $w_s(y)$ was defined in Step 7; $\phi(y)$ = fundamental vibration mode shape of the dam from Fig. 3 or Table 1; $\tilde{M}_1$ and $\tilde{L}_1$ = generalized mass and earthquake coefficient determined in Steps 7 and 8, respectively; and the hydrodynamic pressure term $gp(y,\tilde{T}_r)$ was determined in Step 6; and g = 32.2 feet per squared second.

10. Determine by static analysis of the dam subject to equivalent lateral forces $f_1(y)$, from Step 9, applied to the upstream face of the dam, all the response quantities of interest, in particular the stresses throughout the dam. Traditional procedures for design calculations may be used wherein the direct and bending stresses across a horizontal section are computed by elementary formulas for stresses in beams.*

*However, the beam theory overestimates the stresses near the sloped downstream face by a factor that depends on this slope and the heightwise distribution of equivalent lateral forces. A correction factor is recommended in the next section for the sloping part of the downstream face.

Part II: The earthquake forces and stresses due to the higher vibration modes can be detetermined approximately for purposes of preliminary design by the following computational steps:

11. Compute $f_{sc}(y)$, the lateral forces associated with the higher vibration modes from Eq. 9, in which M_1 and L_1 were determined in Steps 7 and 8, respectively; $g\,p_0(y)$ is determined from Fig. 13 or Table 8; B_1 is computed from Eq. 10; and a_g is the maximum ground acceleration, in feet per squared second, of the design earthquake. If $H/H_s < 0.5$, computation of $p_0(y)$ may be avoided by using $p_0(y) \approx 0$ and hence $B_1 \approx 0$.

12. Determine by static analysis of the dam subjected to the equivalent lateral forces $f_{sc}(y)$, from Step 11, applied to the upstream face of the dam dam, all the response quantities of interest, in particular the stresses throughout the dam. The stress analysis may be carried out by the procedures mentioned in Step 10.

Part III: The total earthquake forces and stresses in the dam are determined by the following computational step:

13. Compute the total value of any response quantity by Eq. 12, in which r_1 and r_{sc} are values of the response quantity determined in Steps 10 and 11 associated with the fundamental and higher vibration modes, respectively, and r_{st} is its initial value prior to the earthquake due to various loads, including the self-weight of the dam, hydrostatic pressure, and thermal effects.

Use of Metric Units

Because the standard values for most quantities required in the simplified analysis procedure are presented in non-dimensional form, implementation of the procedure in metric units is straightforward. The few expressions and data requiring conversion to metric units are noted next:

1. The fundamental vibration T_1 of the dam on rigid foundation rock with empty reservoir (Step 1), in seconds, is given by:

$$T_1 = 0.33\frac{H_s}{\sqrt{E_s}} \quad \text{if} \quad H_s \geq 300\ \text{ft} \tag{14a}$$

$$T_1 = 0.34\frac{H_s}{\sqrt{E_s}} \quad \text{if} \quad H_s < 300\ \text{ft} \tag{14b}$$

   where H_s is the total (monolith plus pier) height of the dam in meters; and E_s is the Young's modulus of elasticity of the dam concrete in mega-Pascals.

2. The period ratio R_r and added damping ratio ξ_r due to dam-water interaction presented in Figs. 4 to 7 and Tables 2 and 3 is for specified values of E_s in psi which should be converted to mega-Pascals as follows: 1 million psi = 7 thousand mega-Pascals.

3. Where required in the calculations, the unit weight of water w = 9.81 kilo-Newtons per cubic meter; the acceleration due to gravity g = 9.81 meters per squared second; and the velocity of pressure waves in water C = 1440 meters per second.

EXAMPLE ANALYSES

System and Ground Motion

The tallest gated spillway monolith of Pine Flat Dam is shown in Fig. 14. In accordance with earlier conclusions, the effects of the gate, bridge, and foot bucket are neglected in this simplified analysis. The total (monolith plus pier) height of the dam $H_s = 400$ ft; monolith thickness = 50 ft, pier thickness = 8 ft, and the ratio of the two is 0.16; modulus of elasticity of concrete, $E_s = 3.25 \times 10^6$ psi; unit weight of concrete = 155 pcf; damping ratio, $\xi_1 = 5\%$; modulus of elasticity of foundation rock, $E_f = 3.25 \times 10^6$ psi; constant hysteretic damping coefficient of foundation rock, $\eta_f = 0.10$; depth of water, $H = 381$ ft; and, at the reservoir bottom, the wave reflection coefficient, $\alpha = 0.5$.

The dam is analyzed for the earthquake ground motion characterized by the smooth design spectrum of Fig. 15, scaled by a factor of 0.25. The spectrum of Fig. 15 is developed by well established procedures [14] for excitations with maximum ground acceleration a_g, velocity v_g, and displacement u_g of $1g$, 48 in./sec, and 36 in., respectively. Amplification factors for the acceleration-controlled, velocity-controlled, and displacement-controlled regions of the spectrum were taken from Ref. 14 for 84.1 percentile response.

Computation of Earthquake Forces

The dam is analyzed by the simplified analysis procedure for the four cases listed in Table 9. Implementation of the step-by-step analysis procedure in the preceding section is summarized next with additional details available in Appendix B; all computations are performed for the equivalent two-dimensional system of unit thickness (see page 2) representing the monolith and pier. Because the height H_s of Pine Flat Dam exceeds 300 ft, all the parameters in the subsequent computations are obtained from tables and figures presented for "higher" dams:

1. For $E_s = 3.25 \times 10^6$ psi and $H_s = 400$ ft, from Eq. 6a, $T_1 = (1.2)(400)/\sqrt{3.25 \times 10^6} = 0.266$ sec.

2. For $E_s = 3.0 \times 10^6$ psi (rounded down from 3.25×10^6 psi), $\alpha = 0.50$ and $H/H_s = 381/400 = 0.95$, Fig. 4(e) or Table 2(e) gives $R_r = 1.319$, so $\tilde{T}_r = (1.319)(0.266) = 0.351$ sec.

3. From Eq. 7, $T_1^r = (4)(381)/4720 = 0.323$ sec and $R_w = 0.323/0.351 = 0.92$.

4. For $E_f/E_s = 1$, Fig. 8 or Table 4 gives $R_f = 1.224$, so $\tilde{T}_1 = (1.224)(0.266) = 0.326$ sec for Case 3, and $\tilde{T}_1 = (1.224)(0.351) = 0.429$ sec for Case 4.

5. For Cases 2 and 4, $\xi_r = 0.046$ from Fig. 6(e) or Table 2(b) for $E_s = 3.0 \times 10^6$ psi (rounded down from 3.25×10^6 psi), $\alpha = 0.50$, and $H/H_s = 0.95$. For Cases 3 and 4, $\xi_f = 0.091$ from

Fig. 9 or Table 4 for $E_f/E_s = 1$ and $\eta_f = 0.10$. With $\xi_1 = 0.05$, Eq. 5 gives: $\tilde{\xi}_1 = (0.05)/(1.319) + 0.046 = 0.084$ for Case 2; $\tilde{\xi}_1 = (0.05)/(1.224)^3 + 0.091 = 0.118$ for Case 3; and $\tilde{\xi}_1 = (0.05)/[(1.319)(1.224)^3] + 0.046 + 0.091 = 0.158$ for Case 4.

6. The values of $gp(y)$ are obtained at fifteen levels (Fig. 16) from Fig. 12(d) or Table 6(d) for $R_w = 0.90$ (by rounding $R_w = 0.92$ from Step 3 and $\alpha = 0.50$, and multiplied by $(0.0624)(381)(0.95)^2 = 21.5$ kip/ft.

7. Evaluating Eq. 2b in discrete form gives $M_1 =$ (1/g)(559 kip). From Eq. 8a, $\tilde{M}_1 = (1.319)^2(1/g)(559) = (973\text{ kip})/g$.

8. Equation 3b in discrete form gives $L_1 = (1623\text{ kip})/g$. From Table 7(b), $A_p = 0.351$ for $R_w = 0.90$ and $\alpha = 0.50$. Equation 8b then gives $\tilde{L}_1 = 1623/g + [(0.0624)(381)^2/2g](0.95)^2(0.351) = (3059\text{ kip})/g$. Consequently, for Cases 1 and 3, $L_1/M_1 = 1623/559 = 2.90$, and for Cases 2 and 4, $\tilde{L}_1/\tilde{M}_1 = 3059/973 = 3.14$.

9. For each of the four cases, Eq. 1 was evaluated to obtain the equivalent lateral forces $f_1(y)$ at fifteen locations along the height of the dam (Fig. 16), including the top and bottom, by substituting values for the quantities computed in the preceding steps; computing the weight $w_s(y)$ per unit height of the monolith from the monolith dimensions (Fig. 14) and the unit weight of concrete; by computing the weight $w_s(y)$ per unit height of the pier from the pier dimensions and the unit weight of concrete, multiplied by 0.16; and by substituting $\phi(y)$ from Fig. 3 or Table 1 and the $S_a(\tilde{T}_1, \tilde{\xi}_1)$ from Fig. 15 corresponding to $\tilde{T}_1$ and $\tilde{\xi}_1$ obtained in Steps 4 and 5 (Table 9). The resulting equivalent lateral forces $f_1(y)$ are presented in Table 10 for each case.

10. The static stress analysis of the dam subjected to the equivalent lateral forces $f_1(y)$, from Step 9, applied to the upstream face of the dam, is described in the next subsection, leading to response value r_1 at a particular location in the dam.

11. For each of the four cases, Eq. 9 was evaluated to obtain the equivalent lateral forces $f_{sc}(y)$ at fifteen locations along the height of the dam (Fig. 16), including the top and bottom, by substituting numerical values for the quantities computed in the preceding steps; obtaining $g\,p_0(y)$ from Fig. 13 or Table 8; using Eq. 10 to compute $B_1 = 0.25[(0.0624)(381)^2/2g](0.95)^2 =$ (1027 kip)/g, leading to $B_1/M_1 =$ 1027/559 = 1.837; and substituting $a_g =$ 0.25 g. The resulting equivalent lateral forces $f_{sc}(y)$ are presented in Table 10 for each case.

12. The static stress analysis of the dam subjected to the equivalent lateral forces $f_{sc}(y)$, from Step 11, applied to the upstream face of the dam, is described in the next subsection, leading to response value r_{sc} at a particular location in the dam.

13. Compute the maximum total value of any response quantity by combining r_1 from Step 10, r_{sc} from Step 12, and r_{st}, the initial value prior to the earthquake, according to Eq. 12; this is described further in the next subsection.

Computation of Stresses

The equivalent lateral earthquake forces $f_1(y)$ and $f_{sc}(y)$ representing the maximum effects of the fundamental and higher vibration modes, respectively, were computed in Steps 9 and 11. Dividing the dam into fourteen blocks shown in Fig. 16, each of these sets of distributed forces is replaced by statically equivalent concentrated forces at the centroids of the blocks. Considering the dam monolith to be a cantilever beam, the vertical bending stresses are computed at the bottom of the blocks of the monolith only (not the pier) using elementary formulas for stresses in beams. The resulting vertical bending stresses due to the fundamental vibration mode and the higher vibration modes are presented in Table 11 for the four analysis cases. Also included are the combined values obtained by the SRSS combination rule. These stresses occur at the upstream face when the earthquake forces act in the downstream direction, and at the downstream face when the earthquake forces act in the upstream direction. In this simple stress analysis the foundation rock is implicitly assumed to be rigid. These computations are implemented by a modified version of the computer program presented in Ref. 2. A description and listing of the computer program is included in this report (Appendix C).

This procedure for computing the stresses is not implemented for the pier. Instead, it should be analyzed as a reinforced concrete structure for the lateral forces computed above. Furthermore, it may be necessary to include the effects of ground motion along the dam axis in the pier analysis.

The vertical bending stresses can be transformed to principal stresses, as described in Appendix C of Ref. 2. Because the upstream face of Pine Flat Dam is nearly vertical and the effects of the tail water at the downstream face are negligible, as shown in Appendix C of Ref. 2, the principal stresses σ_1 and σ_{sc} at any location in the dam due to the forces $f_1(y)$ and $f_{sc}(y)$, respectively, may be combined using the SRSS combination rule, Eq. 11 [2].

As shown in Refs. 2 and 3 for nonoverflow monoliths, while the simplified procedure provides excellent estimates of the maximum stress on the upstream face, at the same time it significantly overestimates the maximum stress on the downstream face. This discrepancy is due primarily to the limitations of elementary beam theory in predicting stresses near sloped faces. The beam theory overestimates the stresses near the sloped downstream face by a factor that depends on this slope and the heightwise distribution of equivalent lateral forces. Based on a comparison of results from beam theory and finite element analysis, it is recommended that σ_{y1} and $\sigma_{y_{sc}}$ computed at the sloping part of the downstream face by beam theory (Steps 10 and 12) should be multiplied by a factor of 0.8. Similarly, the beam theory is incapable of reproducing the stress concentration in the heel area of dams predicted by the refined analysis [2,3]; so the stresses in that area are underestimated. These limitations can be overcome by using the finite element method for static analysis of the dam in Steps 10 and 12 of "Computational Steps."

CONCLUSION

A simplified procedure was presented in 1986 [2] for earthquake analysis of concrete gravity dams. Developed in a form appropriate for preliminary design and safety evaluation of dams, this procedure was presented specifically for nonoverflow monoliths. In this report, the analysis procedure has been extended to gated spillway monoliths, the standard data required for the analysis of such structures has been presented in the form of design charts and tables, and a computer program has been made available to facilitate implementation of the procedure.

This procedure is suitable for stress analysis of the spillway monolith but not for the pier. The latter should be analyzed as a reinforced concrete structure for the lateral forces associated with upstream-downstream ground motion, computed by the procedure presented in this report, and the forces associated with earthquake motion along the dam axis. Evaluation of the latter set of forces is beyond the scope of this report.

Similarly, refined response history analysis procedures [7] are not presently available for analysis of piers. Such procedures could be useful in the seismic safety evaluation of existing dams if the simplified analysis indicates that the piers are likely to be damaged.

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NOTATION

The following symbols are used in this report:

A_p	=	integral of $2gp(\hat{y})/wH$ over depth of the impounded water for $H/H_s = 1$ as listed in Table 7
a_g	=	maximum ground acceleration
B_1	=	defined in Eq. 10
C	=	velocity of pressure waves in water
E_f	=	Young's modulus of elasticity of foundation rock
E_s	=	Young's modulus of elasticity of dam concrete
F_{st}	=	$wH^2/2$
$f_1(y)$	=	equivalent lateral forces on the upstream face of the dam due to the fundamental vibration mode, as defined in Eq. 1
$f_{sc}(y)$	=	equivalent lateral forces on the upstream face of the dam due to higher vibration modes, as defined in Eq. 9
g	=	acceleration due to gravity
H	=	depth of impounded water
H_s	=	height of upstream face of dam (monolith plus pier)
L_1	=	integral defined in Eq. 3b
$\tilde{L}_1$	=	defined in Eq. 3a
M_1	=	integral defined in Eq. 2b
$\tilde{M}_1$	=	defined in Eq. 2a
$p_0(y)$	=	hydrodynamic pressure on a rigid dam with water compressibility neglected

$p(y, \tilde{T}_r)$ = $Re[\bar{p}_1(y, \tilde{T}_r)]$

$\bar{p}_1(y, \tilde{T}_r)$ = complex-valued hydrodynamic pressure on the upstream face due to harmonic acceleration of dam, at period $\tilde{T}_r$, in the fundamental vibration mode

R_f = period lengthening ratio due to foundation-rock flexibility effects

R_r = period lengthening ratio due to hydrodynamic effects

R_w = $T_1^r / \tilde{T}_r$

r_1 = maximum response due to the fundamental vibration mode

r_d = maximum dynamic response

r_{max} = maximum total response of dam

r_{sc} = maximum response due to the higher vibration modes

r_{st} = response due to initial static effects

$S_a(\tilde{T}_1, \tilde{\xi}_1)$ = ordinate of pseudo-acceleration response spectrum for the ground motion evaluated at period $\tilde{T}_1$ and damping ratio $\tilde{\xi}_1$

T_1 = fundamental vibration period of dam on rigid foundation rock with empty reservoir given by Eq. 6

$\tilde{T}_1$ = fundamental resonant period of dam on flexible foundation rock with impounded water given by Eq. 4c

T_1^r = $4H/C$, fundamental vibration period of impounded water

$\tilde{T}_f$ = fundamental resonant period of dam on flexible foundation rock with empty reservoir given by Eq. 4b

$\tilde{T}_r$ = fundamental resonant period of dam on rigid foundation rock with impounded water given by Eq. 4a

W_s = total weight of dam

w = unit weight of water

$w_s(y)$ = weight of dam per unit height; the actual weight of the monolith should be used, but for the pier it should be divided by the ratio of monolith thickness to pier thickness

y = coordinate along the height of the dam

$\hat{y}$ = y/H

α = wave reflection coefficient for reservoir bottom materials or foundation rock

η_f = constant hysteretic damping factor for foundation rock

ξ_1 = damping ratio of dam on rigid foundation rock with empty reservoir

$\tilde{\xi}_1$ = damping ratio for dam on flexible foundation rock with impounded water

ξ_f = added damping ratio due to foundation-rock flexibility effects

ξ_r = added damping ratio due to hydrodynamic effects

$\phi(y)$ = fundamental vibration mode shape of dam at upstream face

TABLES

Table 1(a) -- Standard Fundamental Mode Shape of Vibration for Spillways of Concrete Gravity Dams -- *Higher Dams*

y/H_s	$\phi(y)$
1.0	1.000
.95	.909
.90	.816
.85	.725
.80	.646
.75	.572
.70	.504
.65	.440
.60	.381
.55	.327
.50	.277
.45	.232
.40	.192
.35	.155
.30	.123
.25	.094
.20	.070
.15	.048
.10	.030
.05	.016
0.	0.

Table 1(b) -- Standard Fundamental Mode Shape of Vibration for Spillways of Concrete Gravity Dams -- *Lower Dams*

y/H_s	$\phi(y)$
1.0	1.000
.95	.914
.90	.825
.85	.738
.80	.654
.75	.574
.70	.499
.65	.440
.60	.386
.55	.334
.50	.285
.45	.239
.40	.197
.35	.160
.30	.126
.25	.096
.20	.070
.15	.048
.10	.030
.05	.015
0.	0.

Table 2(a) -- Standard Values for R_r and ξ_r, the Period Lengthening Ratio and Added Damping Ratio due to Hydrodynamic Effects, for Modulus of Elasticity of Concrete, E_s = 5, 4.5 and 4 million psi. -- *Higher Dams*

H/H_s	α	$E_s = 5 \times 10^6$ psi		$E_s = 4.5 \times 10^6$ psi		$E_s = 4 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r	R_r	ξ_r
1.0	1.0	1.642	0.	1.582	0.	1.524	0.
	.9	1.653	0.062	1.590	0.049	1.529	0.036
	.75	1.650	0.073	1.585	0.070	1.527	0.061
	.5	1.502	0.084	1.484	0.079	1.458	0.072
	.25	1.364	0.075	1.366	0.071	1.368	0.065
	.0	1.333	0.055	1.332	0.052	1.330	0.048
.95	1.0	1.548	0.	1.488	0.	1.433	0.
	.9	1.560	0.062	1.497	0.051	1.439	0.037
	.75	1.553	0.068	1.493	0.065	1.435	0.058
	.5	1.350	0.083	1.368	0.075	1.359	0.067
	.25	1.277	0.066	1.280	0.062	1.284	0.057
	.0	1.258	0.047	1.256	0.044	1.255	0.041
.90	1.0	1.460	0.	1.401	0.	1.348	0.
	.9	1.471	0.059	1.410	0.049	1.353	0.036
	.75	1.460	0.061	1.401	0.060	1.348	0.054
	.5	1.235	0.075	1.267	0.068	1.269	0.060
	.25	1.208	0.055	1.212	0.052	1.214	0.048
	.0	1.195	0.039	1.195	0.036	1.193	0.034
.85	1.0	1.374	0.	1.318	0.	1.267	0.
	.9	1.385	0.053	1.326	0.045	1.276	0.031
	.75	1.368	0.055	1.314	0.054	1.267	0.047
	.5	1.170	0.062	1.192	0.057	1.198	0.050
	.25	1.155	0.044	1.157	0.041	1.159	0.038
	.0	1.146	0.031	1.144	0.029	1.144	0.027
.80	1.0	1.290	0.	1.239	0.	1.196	0.
	.9	1.300	0.046	1.248	0.037	1.203	0.024
	.75	1.277	0.049	1.232	0.046	1.195	0.038
	.5	1.126	0.048	1.139	0.044	1.143	0.038
	.25	1.114	0.034	1.116	0.031	1.117	0.029
	.0	1.106	0.023	1.106	0.022	1.105	0.020
.75	1.0	1.209	0.	1.166	0.	1.134	0.
	.9	1.220	0.037	1.175	0.027	1.139	0.016
	.75	1.188	0.042	1.159	0.036	1.134	0.027
	.5	1.094	0.036	1.100	0.032	1.101	0.027
	.25	1.082	0.024	1.083	0.023	1.085	0.020
	.0	1.076	0.017	1.076	0.016	1.075	0.015

Table 2(a) -- Continued

H/H_s	α	$E_s = 5 \times 10^6$ psi		$E_s = 4.5 \times 10^6$ psi		$E_s = 4 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r	R_r	ξ_r
.70	1.0	1.135	0.	1.104	0.	1.086	0.
	.9	1.144	0.026	1.110	0.016	1.089	0.008
	.75	1.117	0.032	1.101	0.024	1.087	0.016
	.5	1.070	0.024	1.072	0.021	1.071	0.017
	.25	1.058	0.017	1.059	0.015	1.059	0.014
	.0	1.054	0.012	1.054	0.011	1.054	0.010
.65	1.0	1.074	0.	1.060	0.	1.054	0.
	.9	1.081	0.013	1.064	0.007	1.055	0.004
	.75	1.071	0.018	1.062	0.012	1.054	0.008
	.5	1.049	0.015	1.049	0.012	1.047	0.010
	.25	1.041	0.011	1.042	0.010	1.042	0.009
	.0	1.037	0.008	1.037	0.007	1.037	0.007
.60	1.0	1.040	0.	1.036	0.	1.033	0.
	.9	1.042	0.004	1.037	0.002	1.034	0.001
	.75	1.041	0.008	1.036	0.005	1.034	0.003
	.5	1.033	0.008	1.032	0.007	1.031	0.005
	.25	1.028	0.007	1.028	0.006	1.028	0.005
	.0	1.026	0.005	1.026	0.005	1.026	0.004
.55	1.0	1.024	0.	1.022	0.	1.020	0.
	.9	1.024	0.001	1.023	0.001	1.022	0.001
	.75	1.024	0.003	1.023	0.002	1.022	0.002
	.5	1.022	0.004	1.020	0.003	1.020	0.003
	.25	1.019	0.004	1.018	0.003	1.018	0.003
	.0	1.017	0.003	1.017	0.003	1.017	0.003
.50	1.0	1.014	0.	1.013	0.	1.013	0.
	.9	1.014	0.000	1.013	0.000	1.013	0.000
	.75	1.014	0.001	1.014	0.001	1.013	0.001
	.5	1.013	0.002	1.013	0.002	1.013	0.001
	.25	1.012	0.002	1.012	0.002	1.012	0.002
	.0	1.011	0.002	1.011	0.002	1.011	0.002

Table 2(b) -- Standard Values for R_r and ξ_r, the Period Lengthening Ratio and Added Damping Ratio due to Hydrodynamic Effects, for Modulus of Elasticity of Concrete, E_s = 3.5 and 3 million psi. -- *Higher Dams*

H/H_s	α	$E_s = 3.5 \times 10^6$ psi		$E_s = 3 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
1.0	1.0	1.475	0.	1.433	0.
	.9	1.477	0.024	1.435	0.015
	.75	1.475	0.048	1.433	0.034
	.5	1.433	0.062	1.407	0.051
	.25	1.364	0.058	1.361	0.051
	.0	1.326	0.045	1.325	0.042
.95	1.0	1.383	0.	1.344	0.
	.9	1.389	0.024	1.348	0.014
	.75	1.385	0.046	1.346	0.032
	.5	1.339	0.058	1.319	0.046
	.25	1.282	0.051	1.279	0.045
	.0	1.253	0.039	1.252	0.035
.90	1.0	1.302	0.	1.267	0.
	.9	1.307	0.022	1.271	0.013
	.75	1.304	0.041	1.269	0.028
	.5	1.259	0.050	1.245	0.039
	.25	1.214	0.043	1.212	0.037
	.0	1.192	0.032	1.191	0.029
.85	1.0	1.229	0.	1.201	0.
	.9	1.233	0.019	1.203	0.010
	.75	1.230	0.035	1.202	0.023
	.5	1.192	0.041	1.183	0.031
	.25	1.160	0.034	1.159	0.029
	.0	1.143	0.025	1.143	0.023
.80	1.0	1.166	0.	1.144	0.
	.9	1.170	0.013	1.147	0.007
	.75	1.167	0.026	1.146	0.016
	.5	1.139	0.031	1.133	0.023
	.25	1.117	0.025	1.116	0.022
	.0	1.105	0.019	1.105	0.017
.75	1.0	1.114	0.	1.101	0.
	.9	1.116	0.008	1.103	0.004
	.75	1.115	0.017	1.101	0.011
	.5	1.099	0.021	1.094	0.016
	.25	1.083	0.018	1.083	0.015
	.0	1.075	0.014	1.075	0.013

Table 2(b) -- Continued

H/H_s	α	$E_s = 3.5 \times 10^6$ psi		$E_s = 3 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
.70	1.0	1.075	0.	1.068	0.
	.9	1.076	0.004	1.070	0.003
	.75	1.076	0.010	1.070	0.006
	.5	1.068	0.013	1.065	0.010
	.25	1.059	0.012	1.058	0.010
	.0	1.054	0.009	1.054	0.009
.65	1.0	1.048	0.	1.046	0.
	.9	1.049	0.002	1.045	0.001
	.75	1.049	0.005	1.044	0.004
	.5	1.046	0.008	1.041	0.006
	.25	1.041	0.008	1.038	0.007
	.0	1.037	0.006	1.035	0.006
.60	1.0	1.031	0.	1.029	0.
	.9	1.031	0.001	1.029	0.001
	.75	1.032	0.002	1.028	0.002
	.5	1.030	0.004	1.027	0.003
	.25	1.028	0.005	1.025	0.004
	.0	1.026	0.004	1.023	0.004
.55	1.0	1.020	0.	1.018	0.
	.9	1.020	0.000	1.018	0.000
	.75	1.020	0.001	1.018	0.001
	.5	1.019	0.002	1.017	0.002
	.25	1.018	0.003	1.016	0.002
	.0	1.017	0.002	1.015	0.002
.50	1.0	1.013	0.	1.011	0.
	.9	1.013	0.000	1.011	0.000
	.75	1.013	0.001	1.011	0.000
	.5	1.013	0.001	1.010	0.001
	.25	1.012	0.001	1.010	0.001
	.0	1.011	0.001	1.009	0.001

Table 2(c) -- Standard Values for R_r and ξ_r, the Period Lengthening Ratio and Added Damping Ratio due to Hydrodynamic Effects, for Modulus of Elasticity of Concrete, E_s = 2.5 and 2 million psi.
Higher Dams

H/H_s	α	$E_s = 2.5 \times 10^6$ psi		$E_s = 2 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
1.0	1.0	1.399	0.	1.384	0.
	.9	1.401	0.009	1.384	0.006
	.75	1.401	0.023	1.380	0.015
	.5	1.385	0.039	1.365	0.029
	.25	1.353	0.043	1.341	0.037
	.0	1.323	0.038	1.319	0.035
.95	1.0	1.314	0.	1.301	0.
	.9	1.316	0.009	1.301	0.005
	.75	1.316	0.021	1.297	0.013
	.5	1.302	0.035	1.284	0.026
	.25	1.274	0.037	1.264	0.032
	.0	1.250	0.032	1.246	0.030
.90	1.0	1.242	0.	1.230	0.
	.9	1.244	0.007	1.230	0.004
	.75	1.242	0.018	1.227	0.011
	.5	1.232	0.029	1.217	0.022
	.25	1.209	0.031	1.201	0.026
	.0	1.191	0.026	1.187	0.025
.85	1.0	1.181	0.	1.172	0.
	.9	1.182	0.006	1.172	0.003
	.75	1.182	0.014	1.170	0.009
	.5	1.172	0.023	1.162	0.017
	.25	1.156	0.024	1.150	0.021
	.0	1.142	0.021	1.140	0.019
.80	1.0	1.131	0.	1.125	0.
	.9	1.133	0.004	1.125	0.002
	.75	1.133	0.010	1.123	0.006
	.5	1.126	0.017	1.118	0.012
	.25	1.115	0.018	1.110	0.015
	.0	1.104	0.016	1.102	0.015
.75	1.0	1.095	0.	1.088	0.
	.9	1.095	0.003	1.088	0.002
	.75	1.093	0.007	1.087	0.004
	.5	1.087	0.012	1.084	0.008
	.25	1.079	0.014	1.079	0.011
	.0	1.073	0.012	1.073	0.011

Table 2(c) -- Continued

H/H_s	α	$E_s = 2.5 \times 10^6$ psi		$E_s = 2 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
.70	1.0	1.065	0.	1.061	0.
	.9	1.064	0.002	1.061	0.001
	.75	1.063	0.004	1.060	0.003
	.5	1.060	0.008	1.058	0.005
	.25	1.055	0.009	1.055	0.007
	.0	1.051	0.008	1.051	0.007
.65	1.0	1.043	0.	1.040	0.
	.9	1.043	0.001	1.040	0.001
	.75	1.042	0.002	1.040	0.002
	.5	1.040	0.005	1.039	0.003
	.25	1.038	0.006	1.037	0.005
	.0	1.035	0.006	1.035	0.005
.60	1.0	1.027	0.	1.026	0.
	.9	1.027	0.001	1.026	0.000
	.75	1.027	0.001	1.026	0.001
	.5	1.026	0.003	1.026	0.002
	.25	1.025	0.003	1.025	0.003
	.0	1.023	0.004	1.023	0.003
.55	1.0	1.017	0.	1.016	0.
	.9	1.017	0.000	1.016	0.000
	.75	1.017	0.001	1.016	0.001
	.5	1.017	0.001	1.016	0.001
	.25	1.016	0.002	1.016	0.002
	.0	1.015	0.002	1.015	0.002
.50	1.0	1.010	0.	1.010	0.
	.9	1.010	0.000	1.010	0.000
	.75	1.010	0.000	1.010	0.000
	.5	1.010	0.001	1.010	0.001
	.25	1.010	0.001	1.010	0.001
	.0	1.009	0.001	1.009	0.001

Table 3(a) -- Standard Values for R_r and ξ_r, the Period Lengthening Ratio and Added Damping Ratio due to Hydrodynamic Effects, for Modulus of Elasticity of Concrete, E_s = 5, 4.5 and 4 million psi. -- *Lower Dams*

H/H_s	α	$E_s = 5 \times 10^6$ psi		$E_s = 4.5 \times 10^6$ psi		$E_s = 4 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r	R_r	ξ_r
1.0	1.0	1.647	0.	1.600	0.	1.555	0.
	.9	1.653	0.043	1.605	0.031	1.560	0.022
	.75	1.653	0.069	1.603	0.059	1.558	0.048
	.5	1.572	0.081	1.543	0.074	1.518	0.065
	.25	1.454	0.075	1.451	0.070	1.447	0.063
	.0	1.405	0.057	1.403	0.054	1.401	0.050
.95	1.0	1.546	0.	1.497	0.	1.456	0.
	.9	1.553	0.046	1.504	0.034	1.458	0.024
	.75	1.550	0.067	1.499	0.059	1.456	0.048
	.5	1.456	0.077	1.435	0.070	1.412	0.062
	.25	1.350	0.068	1.350	0.063	1.348	0.057
	.0	1.316	0.050	1.314	0.047	1.312	0.044
.90	1.0	1.451	0.	1.405	0.	1.362	0.
	.9	1.458	0.047	1.410	0.034	1.368	0.023
	.75	1.454	0.063	1.407	0.056	1.364	0.045
	.5	1.346	0.072	1.335	0.065	1.321	0.056
	.25	1.266	0.059	1.266	0.055	1.266	0.049
	.0	1.241	0.042	1.239	0.040	1.238	0.037
.85	1.0	1.361	0.	1.318	0.	1.280	0.
	.9	1.370	0.044	1.325	0.032	1.285	0.021
	.75	1.362	0.057	1.318	0.051	1.282	0.039
	.5	1.250	0.063	1.250	0.056	1.241	0.048
	.25	1.198	0.048	1.199	0.045	1.199	0.041
	.0	1.179	0.034	1.179	0.032	1.178	0.030
.80	1.0	1.277	0.	1.238	0.	1.208	0.
	.9	1.287	0.038	1.245	0.027	1.212	0.016
	.75	1.276	0.050	1.238	0.043	1.209	0.032
	.5	1.179	0.052	1.181	0.045	1.175	0.038
	.25	1.144	0.038	1.146	0.035	1.146	0.031
	.0	1.133	0.026	1.131	0.025	1.131	0.023
.75	1.0	1.201	0.	1.168	0.	1.146	0.
	.9	1.208	0.030	1.174	0.019	1.149	0.011
	.75	1.195	0.041	1.168	0.032	1.147	0.023
	.5	1.127	0.039	1.129	0.033	1.125	0.027
	.25	1.105	0.028	1.105	0.026	1.105	0.023
	.0	1.095	0.019	1.095	0.018	1.094	0.017

Table 3(a) -- Continued

H/H_s	α	$E_s = 5 \times 10^6$ psi		$E_s = 4.5 \times 10^6$ psi		$E_s = 4 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r	R_r	ξ_r
.70	1.0	1.131	0.	1.111	0.	1.098	0.
	.9	1.138	0.020	1.115	0.011	1.100	0.006
	.75	1.129	0.029	1.111	0.021	1.099	0.014
	.5	1.091	0.026	1.089	0.022	1.087	0.018
	.25	1.075	0.019	1.075	0.018	1.075	0.016
	.0	1.067	0.014	1.067	0.013	1.067	0.012
.65	1.0	1.079	0.	1.070	0.	1.064	0.
	.9	1.083	0.009	1.072	0.005	1.065	0.003
	.75	1.080	0.016	1.071	0.011	1.064	0.007
	.5	1.063	0.016	1.060	0.013	1.058	0.011
	.25	1.053	0.013	1.052	0.011	1.052	0.010
	.0	1.047	0.009	1.047	0.009	1.047	0.008
.60	1.0	1.046	0.	1.043	0.	1.041	0.
	.9	1.048	0.003	1.044	0.002	1.041	0.001
	.75	1.047	0.007	1.044	0.005	1.041	0.004
	.5	1.041	0.009	1.040	0.007	1.038	0.006
	.25	1.035	0.008	1.035	0.007	1.035	0.006
	.0	1.032	0.006	1.032	0.006	1.032	0.005
.55	1.0	1.028	0.	1.027	0.	1.026	0.
	.9	1.029	0.001	1.027	0.001	1.026	0.001
	.75	1.029	0.003	1.027	0.002	1.026	0.002
	.5	1.027	0.004	1.026	0.004	1.025	0.003
	.25	1.024	0.004	1.024	0.004	1.024	0.003
	.0	1.022	0.004	1.022	0.003	1.022	0.003
.50	1.0	1.017	0.	1.016	0.	1.016	0.
	.9	1.017	0.001	1.016	0.000	1.016	0.000
	.75	1.017	0.001	1.016	0.001	1.016	0.001
	.5	1.016	0.002	1.016	0.002	1.016	0.001
	.25	1.015	0.002	1.015	0.002	1.015	0.002
	.0	1.014	0.002	1.014	0.002	1.014	0.002

Table 3(b) -- Standard Values for R_r and ξ_r, the Period Lengthening Ratio and Added Damping Ratio due to Hydrodynamic Effects, for Modulus of Elasticity of Concrete, E_s = 3.5 and 3 million psi. -- *Lower Dams*

H/H_s	α	$E_s = 3.5 \times 10^6$ psi		$E_s = 3 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
1.0	1.0	1.520	0.	1.486	0.
	.9	1.522	0.015	1.488	0.011
	.75	1.520	0.036	1.488	0.026
	.5	1.493	0.055	1.471	0.044
	.25	1.441	0.056	1.433	0.049
	.0	1.399	0.047	1.395	0.043
.95	1.0	1.418	0.	1.389	0.
	.9	1.421	0.016	1.391	0.010
	.75	1.421	0.035	1.389	0.025
	.5	1.393	0.051	1.374	0.041
	.25	1.344	0.051	1.339	0.044
	.0	1.311	0.041	1.309	0.038
.90	1.0	1.330	0.	1.304	0.
	.9	1.332	0.015	1.306	0.009
	.75	1.332	0.033	1.304	0.023
	.5	1.304	0.046	1.289	0.036
	.25	1.264	0.044	1.259	0.038
	.0	1.236	0.035	1.236	0.032
.85	1.0	1.252	0.	1.230	0.
	.9	1.255	0.013	1.232	0.008
	.75	1.253	0.028	1.230	0.019
	.5	1.229	0.039	1.218	0.030
	.25	1.198	0.036	1.195	0.031
	.0	1.178	0.028	1.177	0.026
.80	1.0	1.185	0.	1.168	0.
	.9	1.188	0.010	1.170	0.006
	.75	1.186	0.022	1.170	0.014
	.5	1.168	0.030	1.160	0.023
	.25	1.146	0.028	1.143	0.024
	.0	1.130	0.022	1.130	0.020
.75	1.0	1.130	0.	1.120	0.
	.9	1.133	0.007	1.121	0.004
	.75	1.131	0.015	1.121	0.010
	.5	1.120	0.021	1.115	0.016
	.25	1.105	0.020	1.104	0.017
	.0	1.094	0.016	1.094	0.015

Table 3(b) -- Continued

H/H_s	α	$E_s = 3.5 \times 10^6$ psi		$E_s = 3 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
.70	1.0	1.089	0.	1.082	0.
	.9	1.091	0.004	1.083	0.002
	.75	1.089	0.009	1.083	0.006
	.5	1.083	0.014	1.080	0.010
	.25	1.074	0.014	1.073	0.012
	.0	1.067	0.011	1.067	0.010
.65	1.0	1.059	0.	1.056	0.
	.9	1.059	0.002	1.056	0.001
	.75	1.059	0.005	1.055	0.004
	.5	1.056	0.008	1.052	0.007
	.25	1.052	0.009	1.048	0.008
	.0	1.047	0.008	1.044	0.007
.60	1.0	1.038	0.	1.036	0.
	.9	1.038	0.001	1.036	0.001
	.75	1.038	0.003	1.035	0.002
	.5	1.037	0.005	1.034	0.004
	.25	1.034	0.005	1.032	0.005
	.0	1.032	0.005	1.029	0.005
.55	1.0	1.025	0.	1.022	0.
	.9	1.025	0.001	1.022	0.000
	.75	1.025	0.001	1.022	0.001
	.5	1.025	0.002	1.022	0.002
	.25	1.023	0.003	1.020	0.003
	.0	1.022	0.003	1.019	0.003
.50	1.0	1.015	0.	1.013	0.
	.9	1.015	0.000	1.013	0.000
	.75	1.015	0.001	1.013	0.001
	.5	1.015	0.001	1.013	0.001
	.25	1.015	0.002	1.012	0.002
	.0	1.014	0.002	1.012	0.002

Table 3(c) -- Standard Values for R_r and ξ_r, the Period Lengthening Ratio and Added Damping Ratio due to Hydrodynamic Effects, for Modulus of Elasticity of Concrete, E_s = 2.5 and 2 million psi.
Lower Dams

H/H_s	α	$E_s = 2.5 \times 10^6$ psi		$E_s = 2 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
1.0	1.0	1.460	0.	1.449	0.
	.9	1.462	0.007	1.448	0.005
	.75	1.462	0.018	1.446	0.013
	.5	1.451	0.034	1.433	0.026
	.25	1.425	0.041	1.412	0.035
	.0	1.393	0.039	1.387	0.037
.95	1.0	1.364	0.	1.354	0.
	.9	1.366	0.007	1.353	0.004
	.75	1.366	0.017	1.351	0.012
	.5	1.355	0.031	1.340	0.024
	.25	1.333	0.037	1.321	0.032
	.0	1.306	0.035	1.301	0.032
.90	1.0	1.282	0.	1.272	0.
	.9	1.284	0.006	1.272	0.004
	.75	1.284	0.015	1.270	0.010
	.5	1.274	0.027	1.261	0.020
	.25	1.255	0.032	1.246	0.027
	.0	1.235	0.029	1.230	0.027
.85	1.0	1.214	0.	1.205	0.
	.9	1.214	0.005	1.205	0.003
	.75	1.214	0.012	1.203	0.008
	.5	1.206	0.022	1.196	0.017
	.25	1.192	0.026	1.184	0.022
	.0	1.175	0.023	1.172	0.022
.80	1.0	1.156	0.	1.150	0.
	.9	1.157	0.004	1.150	0.002
	.75	1.157	0.009	1.149	0.006
	.5	1.152	0.017	1.144	0.013
	.25	1.142	0.020	1.135	0.017
	.0	1.130	0.018	1.126	0.017
.75	1.0	1.111	0.	1.107	0.
	.9	1.112	0.003	1.107	0.002
	.75	1.112	0.006	1.106	0.004
	.5	1.109	0.012	1.103	0.009
	.25	1.101	0.014	1.097	0.012
	.0	1.094	0.013	1.091	0.012

Table 3(c) -- Continued

H/H_s	α	$E_s = 2.5 \times 10^6$ psi		$E_s = 2 \times 10^6$ psi	
		R_r	ξ_r	R_r	ξ_r
.70	1.0	1.078	0.	1.075	0.
	.9	1.079	0.002	1.075	0.001
	.75	1.079	0.004	1.074	0.003
	.5	1.076	0.008	1.072	0.006
	.25	1.072	0.010	1.069	0.008
	.0	1.066	0.009	1.064	0.009
.65	1.0	1.053	0.	1.051	0.
	.9	1.053	0.001	1.050	0.001
	.75	1.052	0.003	1.050	0.002
	.5	1.050	0.005	1.049	0.004
	.25	1.047	0.007	1.047	0.005
	.0	1.044	0.007	1.044	0.006
.60	1.0	1.034	0.	1.033	0.
	.9	1.034	0.001	1.033	0.000
	.75	1.034	0.001	1.033	0.001
	.5	1.033	0.003	1.032	0.002
	.25	1.031	0.004	1.031	0.003
	.0	1.029	0.004	1.029	0.004
.55	1.0	1.021	0.	1.021	0.
	.9	1.022	0.000	1.021	0.000
	.75	1.021	0.001	1.021	0.001
	.5	1.021	0.002	1.021	0.001
	.25	1.020	0.002	1.020	0.002
	.0	1.019	0.003	1.019	0.002
.50	1.0	1.013	0.	1.013	0.
	.9	1.013	0.000	1.013	0.000
	.75	1.013	0.000	1.013	0.000
	.5	1.013	0.001	1.013	0.001
	.25	1.012	0.001	1.012	0.001
	.0	1.012	0.002	1.012	0.001

Table 4 -- Standard Values for R_f and ξ_f, the Period Lengthening Ratio and the Added Damping Ratio, due to Dam-Foundation Rock Interaction -- *Higher Dams*

E_f/E_s	η_f = 0.01		η_f = 0.10		η_f = 0.25		η_f = 0.50	
	R_f	ξ_f	R_f	ξ_f	R_f	ξ_f	R_f	ξ_f
5.0	1.058	0.017	1.055	0.020	1.050	0.026	1.040	0.032
4.5	1.064	0.019	1.060	0.022	1.055	0.029	1.044	0.036
4.0	1.071	0.022	1.067	0.025	1.061	0.032	1.048	0.040
3.5	1.081	0.025	1.076	0.029	1.068	0.037	1.054	0.045
3.0	1.093	0.028	1.087	0.033	1.078	0.042	1.062	0.052
2.5	1.110	0.034	1.102	0.040	1.092	0.050	1.072	0.061
2.0	1.134	0.041	1.124	0.049	1.111	0.062	1.087	0.075
1.5	1.172	0.053	1.159	0.064	1.142	0.080	1.110	0.097
1.4	1.182	0.057	1.169	0.068	1.150	0.085	1.116	0.103
1.3	1.194	0.060	1.180	0.072	1.160	0.090	1.123	0.109
1.2	1.207	0.065	1.192	0.078	1.171	0.096	1.131	0.117
1.1	1.221	0.069	1.207	0.084	1.183	0.104	1.140	0.126
1.0	1.240	0.075	1.224	0.091	1.198	0.112	1.151	0.136
0.9	1.261	0.082	1.244	0.099	1.215	0.122	1.163	0.149
0.8	1.287	0.090	1.269	0.109	1.236	0.134	1.178	0.163
0.7	1.319	0.100	1.299	0.120	1.262	0.149	1.196	0.182
0.6	1.362	0.112	1.338	0.135	1.295	0.167	1.219	0.205
0.5	1.417	0.128	1.389	0.154	1.339	0.190	1.249	0.235
0.4	1.491	0.150	1.462	0.178	1.399	0.221	1.289	0.276
0.3	1.612	0.181	1.572	0.212	1.490	0.265	1.347	0.338
0.2	1.829	0.218	1.764	0.261	1.646	0.333	1.438	0.443

Table 5 -- Standard Values for R_f and ξ_f, the Period Lengthening Ratio and the Added Damping Ratio, due to Dam-Foundation Rock Interaction -- *Lower Dams*

E_f/E_s	η_f = 0.01		η_f = 0.10		η_f = 0.25		η_f = 0.50	
	R_f	ξ_f	R_f	ξ_f	R_f	ξ_f	R_f	ξ_f
5.0	1.056	0.011	1.054	0.017	1.050	0.023	1.040	0.030
4.5	1.062	0.012	1.060	0.019	1.055	0.026	1.044	0.033
4.0	1.070	0.014	1.066	0.021	1.061	0.029	1.049	0.037
3.5	1.079	0.016	1.075	0.024	1.069	0.033	1.055	0.042
3.0	1.091	0.018	1.086	0.028	1.079	0.038	1.063	0.048
2.5	1.108	0.022	1.102	0.033	1.093	0.044	1.074	0.057
2.0	1.133	0.028	1.124	0.041	1.113	0.054	1.090	0.069
1.5	1.172	0.037	1.161	0.054	1.145	0.070	1.115	0.089
1.4	1.183	0.039	1.171	0.057	1.154	0.074	1.121	0.094
1.3	1.195	0.042	1.182	0.061	1.164	0.079	1.129	0.100
1.2	1.209	0.046	1.195	0.065	1.176	0.085	1.138	0.107
1.1	1.225	0.050	1.210	0.070	1.189	0.091	1.148	0.115
1.0	1.244	0.055	1.228	0.076	1.205	0.098	1.160	0.124
0.9	1.266	0.061	1.249	0.083	1.224	0.107	1.175	0.135
0.8	1.293	0.068	1.275	0.091	1.247	0.117	1.192	0.148
0.7	1.325	0.077	1.308	0.100	1.275	0.129	1.213	0.165
0.6	1.366	0.088	1.349	0.112	1.318	0.144	1.240	0.185
0.5	1.421	0.103	1.405	0.127	1.360	0.163	1.276	0.210
0.4	1.505	0.121	1.483	0.146	1.429	0.188	1.327	0.245
0.3	1.643	0.143	1.604	0.172	1.534	0.223	1.402	0.295
0.2	1.870	0.166	1.818	0.208	1.719	0.273	1.533	0.374

Table 6(a) -- Standard Values for the Hydrodynamic Pressure Function $p(\hat{y})$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 1.00$ -- *Higher and Lower Dams*

$\hat{y}=y/H$	Value of $gp(\hat{y})/wH$						
	$R_w \leq 0.5$	$R_w=0.7$	$R_w=0.8$	$R_w=0.85$	$R_w=0.9$	$R_w=0.92$	$R_w=0.93$
1.00	0	0	0	0	0	0	0
.95	.080	.083	.087	.090	.096	.099	.102
.90	.131	.138	.146	.152	.163	.170	.175
.85	.153	.163	.175	.184	.201	.211	.218
.80	.164	.178	.193	.206	.228	.242	.251
.75	.176	.194	.212	.228	.254	.271	.283
.70	.184	.204	.226	.244	.276	.296	.310
.65	.183	.207	.231	.253	.289	.312	.328
.60	.180	.206	.234	.258	.298	.325	.342
.55	.179	.207	.238	.264	.309	.338	.357
.50	.177	.207	.240	.269	.317	.349	.369
.45	.170	.203	.238	.269	.321	.355	.377
.40	.164	.198	.236	.268	.323	.359	.383
.35	.159	.196	.235	.269	.327	.365	.390
.30	.155	.193	.234	.269	.330	.369	.395
.25	.149	.188	.230	.267	.330	.370	.397
.20	.143	.183	.226	.264	.329	.371	.399
.15	.141	.181	.225	.264	.330	.373	.401
.10	.139	.179	.224	.263	.330	.374	.403
.05	.135	.176	.222	.261	.329	.373	.402
0	.133	.175	.220	.260	.327	.372	.401

Table 6(a) -- Continued

$\hat{y}=y/H$	Value of $gp(\hat{y})/wH$					
	R_w=0.94	R_w=0.95	R_w=0.96	R_w=0.97	R_w=0.98	R_w=0.99
1.00	0	0	0	0	0	0
.95	.105	.108	.113	.121	.133	.161
.90	.181	.188	.198	.213	.237	.293
.85	.227	.238	.253	.275	.311	.394
.80	.262	.276	.296	.328	.374	.484
.75	.297	.315	.339	.375	.435	.572
.70	.326	.347	.376	.419	.490	.652
.65	.347	.371	.405	.453	.536	.722
.60	.363	.391	.428	.483	.576	.785
.55	.380	.411	.452	.513	.615	.847
.50	.395	.428	.473	.539	.651	.902
.45	.405	.440	.489	.560	.679	.950
.40	.412	.450	.502	.577	.705	.992
.35	.421	.461	.515	.595	.729	1.032
.30	.428	.469	.526	.609	.749	1.066
.25	.431	.474	.533	.619	.764	1.093
.20	.433	.477	.538	.627	.776	1.115
.15	.436	.482	.543	.634	.787	1.133
.10	.438	.485	.547	.640	.795	1.146
.05	.438	.484	.548	.641	.798	1.152
0	.437	.484	.547	.641	.798	1.154

Table 6(b) -- Standard Values for the Hydrodynamic Pressure Function $p(\hat{y})$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.90$ -- *Higher and Lower Dams*

$\hat{y}=y/H$	Value of $gp(\hat{y})/wH$								
	$R_w \leq 0.5$	$R_w=0.7$	$R_w=0.8$	$R_w=0.9$	$R_w=.95$	$R_w=1.0$	$R_w=1.05$	$R_w=1.1$	$R_w=1.2$
1.00	0	0	0	0	0	0	0	0	0
.95	.080	.083	.087	.095	.103	.104	.078	.071	.069
.90	.131	.138	.145	.161	.178	.178	.127	.114	.108
.85	.152	.163	.174	.198	.222	.223	.147	.127	.119
.80	.164	.178	.193	.223	.256	.257	.156	.130	.118
.75	.176	.193	.211	.249	.289	.290	.165	.133	.118
.70	.184	.204	.225	.270	.318	.318	.170	.131	.114
.65	.183	.206	.230	.282	.337	.337	.166	.122	.102
.60	.180	.206	.233	.290	.352	.352	.160	.109	.087
.55	.179	.207	.236	.300	.368	.368	.155	.099	.074
.50	.176	.207	.239	.308	.381	.381	.149	.088	.060
.45	.170	.202	.237	.311	.389	.388	.139	.074	.044
.40	.164	.198	.234	.312	.396	.394	.129	.059	.027
.35	.159	.195	.233	.315	.403	.400	.121	.047	.013
.30	.155	.192	.231	.317	.409	.405	.113	.036	.001
.25	.149	.187	.228	.316	.411	.407	.104	.024	.000
.20	.143	.182	.224	.315	.413	.407	.095	.013	.000
.15	.140	.180	.223	.316	.415	.408	.089	.005	.000
.10	.138	.179	.222	.316	.416	.408	.084	.000	.000
.05	.135	.176	.219	.314	.415	.406	.078	.000	.000
0	.133	.174	.217	.312	.414	.403	.074	.000	.000

Table 6(c) -- Standard Values for the Hydrodynamic Pressure Function $p(\hat{y})$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.75$ -- *Higher and Lower Dams*

$\hat{y}=y/H$	Value of $gp(\hat{y})/wH$								
	$R_w \leq 0.5$	$R_w=0.7$	$R_w=0.8$	$R_w=0.9$	$R_w=.95$	$R_w=1.0$	$R_w=1.05$	$R_w=1.1$	$R_w=1.2$
1.00	0	0	0	0	0	0	0	0	0
.95	.080	.083	.086	.091	.092	.090	.083	.077	.072
.90	.131	.137	.143	.153	.156	.151	.137	.126	.116
.85	.152	.162	.171	.185	.190	.182	.161	.145	.130
.80	.164	.177	.189	.207	.213	.203	.175	.153	.133
.75	.176	.192	.206	.228	.236	.223	.189	.161	.136
.70	.183	.202	.219	.245	.254	.239	.198	.165	.135
.65	.183	.204	.224	.253	.263	.245	.198	.160	.126
.60	.180	.203	.225	.258	.269	.249	.196	.153	.114
.55	.178	.204	.228	.264	.276	.253	.194	.147	.103
.50	.176	.204	.230	.268	.281	.256	.191	.139	.092
.45	.170	.199	.227	.268	.282	.254	.184	.128	.077
.40	.163	.194	.223	.267	.281	.250	.176	.116	.061
.35	.159	.191	.221	.267	.281	.249	.170	.106	.049
.30	.154	.188	.219	.266	.281	.246	.163	.097	.037
.25	.148	.183	.215	.263	.278	.241	.155	.086	.023
.20	.142	.178	.211	.260	.274	.235	.146	.075	.011
.15	.139	.175	.209	.258	.272	.232	.140	.068	.002
.10	.137	.173	.207	.257	.270	.229	.135	.061	.000
.05	.134	.170	.204	.254	.266	.223	.128	.054	.000
0	.132	.168	.202	.251	.263	.219	.123	.048	.000

Table 6(d) -- Standard Values for the Hydrodynamic Pressure Function $p(\hat{y})$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.50$ -- *Higher and Lower Dams*

$\hat{y}=y/H$	Value of $gp(\hat{y})/wH$								
	$R_w \leq 0.5$	$R_w=0.7$	$R_w=0.8$	$R_w=0.9$	$R_w=.95$	$R_w=1.0$	$R_w=1.05$	$R_w=1.1$	$R_w=1.2$
1.00	0	0	0	0	0	0	0	0	0
.95	.079	.082	.083	.084	.084	.083	.082	.080	.077
.90	.130	.135	.138	.140	.139	.137	.134	.131	.125
.85	.151	.158	.163	.166	.165	.162	.158	.152	.143
.80	.163	.172	.178	.181	.180	.176	.170	.163	.151
.75	.174	.185	.192	.196	.195	.190	.183	.174	.158
.70	.181	.194	.202	.207	.205	.199	.190	.180	.161
.65	.181	.195	.204	.209	.207	.200	.189	.177	.155
.60	.177	.193	.203	.208	.206	.198	.185	.171	.146
.55	.176	.193	.204	.209	.206	.197	.183	.167	.138
.50	.173	.191	.202	.208	.204	.194	.178	.160	.129
.45	.166	.186	.197	.202	.198	.186	.170	.150	.116
.40	.159	.180	.191	.196	.191	.178	.160	.139	.103
.35	.155	.175	.187	.192	.186	.172	.152	.130	.091
.30	.150	.171	.183	.187	.180	.166	.145	.121	.080
.25	.144	.165	.177	.180	.172	.157	.134	.109	.066
.20	.138	.159	.171	.173	.165	.148	.124	.098	.054
.15	.134	.156	.167	.168	.159	.141	.117	.090	.044
.10	.132	.153	.164	.164	.154	.135	.110	.082	.035
.05	.128	.149	.159	.158	.148	.128	.101	.073	.025
0	.126	.146	.156	.153	.142	.121	.094	.065	.017

Table 6(e) -- Standard Values for the Hydrodynamic Pressure Function $p(\hat{y})$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.25$ -- *Higher and Lower Dams*

$\hat{y}=y/H$	Value of $gp(\hat{y})/wH$								
	$R_w \le 0.5$	$R_w=0.7$	$R_w=0.8$	$R_w=0.9$	$R_w=.95$	$R_w=1.0$	$R_w=1.05$	$R_w=1.1$	$R_w=1.2$
1.00	0	0	0	0	0	0	0	0	0
.95	.079	.080	.080	.081	.081	.080	.080	.080	.079
.90	.129	.131	.132	.132	.132	.132	.131	.130	.129
.85	.150	.153	.154	.155	.154	.154	.153	.152	.149
.80	.160	.164	.166	.167	.166	.165	.164	.162	.158
.75	.171	.176	.178	.178	.178	.177	.175	.173	.168
.70	.178	.183	.185	.186	.185	.183	.181	.178	.172
.65	.176	.182	.184	.184	.183	.181	.178	.175	.168
.60	.172	.179	.181	.180	.179	.176	.173	.169	.160
.55	.170	.177	.178	.177	.175	.172	.169	.164	.153
.50	.166	.173	.175	.173	.171	.167	.163	.157	.145
.45	.159	.166	.167	.165	.162	.158	.152	.146	.133
.40	.152	.158	.159	.156	.152	.147	.141	.135	.120
.35	.146	.152	.152	.148	.144	.139	.132	.125	.108
.30	.141	.147	.146	.141	.137	.130	.123	.115	.097
.25	.134	.139	.138	.132	.126	.119	.111	.102	.083
.20	.127	.131	.129	.122	.116	.109	.100	.090	.069
.15	.124	.126	.124	.115	.109	.101	.091	.080	.059
.10	.120	.122	.118	.109	.102	.093	.083	.071	.048
.05	.116	.117	.112	.101	.093	.084	.073	.061	.037
0	.113	.112	.107	.095	.086	.076	.064	.052	.027

Table 6(f) -- Standard Values for the Hydrodynamic Pressure Function $p(\hat{y})$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.00$ -- *Higher and Lower Dams*

$\hat{y}=y/H$	Value of $gp(\hat{y})/wH$								
	$R_w\leq 0.5$	$R_w=0.7$	$R_w=0.8$	$R_w=0.9$	$R_w=.95$	$R_w=1.0$	$R_w=1.05$	$R_w=1.1$	$R_w=1.2$
1.00	0	0	0	0	0	0	0	0	0
.95	.078	.078	.078	.079	.079	.079	.079	.079	.079
.90	.127	.127	.128	.128	.129	.129	.129	.129	.130
.85	.146	.147	.148	.149	.149	.149	.150	.150	.151
.80	.156	.157	.158	.158	.159	.159	.160	.160	.161
.75	.166	.167	.168	.168	.169	.169	.169	.170	.170
.70	.171	.172	.173	.173	.174	.174	.174	.175	.175
.65	.169	.170	.170	.170	.171	.171	.171	.171	.171
.60	.164	.164	.164	.164	.164	.164	.164	.164	.164
.55	.161	.160	.160	.160	.159	.159	.159	.158	.158
.50	.156	.155	.154	.153	.153	.152	.152	.151	.149
.45	.148	.146	.145	.143	.142	.142	.141	.139	.137
.40	.139	.136	.135	.132	.131	.130	.128	.127	.123
.35	.133	.129	.126	.123	.122	.120	.118	.116	.112
.30	.126	.121	.118	.114	.112	.110	.108	.105	.100
.25	.118	.112	.108	.103	.101	.098	.095	.092	.085
.20	.111	.103	.098	.092	.089	.086	.082	.079	.071
.15	.106	.096	.090	.084	.080	.076	.072	.068	.059
.10	.101	.090	.083	.076	.071	.067	.063	.058	.048
.05	.096	.083	.075	.066	.062	.057	.051	.046	.035
0	.092	.077	.068	.058	.053	.047	.042	.036	.023

Table 7(a) -- Standard Values for A_p, the Hydrodynamic Force Coefficient in $\tilde{L}_1$; $\alpha = 1$

R_w	Value of A_p for $\alpha = 1$
0.99	1.608
0.98	1.155
0.97	.955
0.96	.836
0.95	.755
0.94	.695
0.93	.649
0.92	.612
0.90	.555
0.85	.468
0.80	.417
0.70	.358
$\leq$ 0.50	.303

Table 7(b) -- Standard Values for A_p, the Hydrodynamic Force Coefficient in $\tilde{L}_1$; α = 0.90,0.75,0.50,0.25 and 0

R_w	Value of A_p				
	$\alpha = 0.90$	$\alpha = 0.75$	$\alpha = 0.50$	$\alpha = 0.25$	$\alpha = 0$
1.20	.088	.139	.201	.225	.230
1.10	.138	.225	.259	.250	.236
1.05	.247	.319	.292	.261	.239
1.00	.664	.437	.322	.271	.242
0.95	.669	.485	.342	.279	.245
0.90	.537	.464	.351	.286	.247
0.80	.414	.397	.345	.292	.252
0.70	.357	.351	.327	.292	.256
$\leq$ 0.50	.303	.302	.296	.283	.262

Table 8 -- Standard Values for the Hydrodynamic Pressure Function $p_o(\hat{y})$

$\hat{y} = y/H_s$	$gp_o(y)/wH$
1.0	0.
.95	.137
.90	.224
.85	.301
.80	.362
.75	.418
.70	.465
.65	.509
.60	.546
.55	.580
.50	.610
.45	.637
.40	.659
.35	.680
.30	.696
.25	.711
.20	.722
.15	.731
.10	.737
.05	.741
0.	.742

Table 9 -- Pine Flat Dam Analysis Cases, Simplified Procedure Parameters, and Fundamental Mode Properties

Case	Foundation Rock	Water	Parameters				Fundamental Mode Properties		
							Vibration Period in seconds	Damping Ratio	$S_a(\tilde{T}_1, \tilde{\xi}_1)$ in g's
			R_r	R_f	ξ_r	ξ_f	$\tilde{T}_1$	$\tilde{\xi}_1$	
1	rigid	empty	1.0	1.0	0	0	0.266	0.050	0.677
2	flexible	full	1.319	1.0	0.046	0	0.351	0.084	0.542
3	rigid	empty	1.0	1.224	0	0.091	0.326	0.118	0.453
4	flexible	full	1.319	1.224	0.046	0.091	0.429	0.158	0.377

Table 10 -- Equivalent Lateral Earthquake Forces on Pine Flat Dam due to Earthquake Ground Motion Characterized by the Smooth Design Spectrum of Fig. 15, Scaled by a Factor of 0.25

y (ft)	w_s (k/ft)	ϕ	$w_s\phi$ (k/ft)	gp (k/ft)	gp_o (k/ft)	Lateral Forces, in kips per foot							
						Case 1		Case 2		Case 3		Case 4	
						f_1	f_{sc}	f_1	f_{sc}	f_1	f_{sc}	f_1	f_{sc}
400.	0.74	1.0	0.74	0	0.	1.45	-0.35	1.26	-0.69	0.97	-0.35	0.87	-0.69
389.38	0.74	0.95	0.70	0	0.	1.38	-0.33	1.20	-0.65	0.93	-0.33	0.83	-0.65
359.286	1.52	0.81	1.23	1.98	3.54	2.43	-0.52	5.48	-0.20	1.62	-0.52	3.81	-0.20
340.	1.52	0.73	1.10	3.11	5.61	2.17	-0.42	7.16	0.48	1.45	-0.42	4.98	0.48
335.	5.39	0.71	3.81	3.26	6.09	7.48	-1.42	12.0	-1.64	5.00	-1.42	8.36	-1.64
308.85	9.70	0.61	5.87	3.84	8.30	11.5	-1.84	16.5	-2.46	7.72	-1.84	11.5	-2.46
300.	10.90	0.57	6.24	3.99	8.96	12.3	-1.80	17.4	-2.42	8.20	-1.80	12.1	-2.42
280.	13.47	0.50	6.79	4.29	10.3	13.3	-1.56	18.9	-2.11	8.93	-1.56	13.1	-2.11
240.	18.62	0.38	7.09	4.51	12.5	13.9	-0.50	19.7	-0.64	9.33	-0.50	13.7	-0.64
200.	23.72	0.28	6.57	4.51	14.1	12.9	1.16	18.9	1.68	8.64	1.16	13.1	1.68
160.	28.91	0.19	5.55	4.27	15.5	10.9	3.20	16.7	4.51	7.30	3.20	11.6	4.51
120.	34.05	0.12	4.19	4.08	16.4	8.23	5.47	14.1	7.66	5.51	5.47	9.79	7.66
80.	39.20	0.07	2.74	3.75	17.1	5.39	7.81	11.1	10.8	3.61	7.81	7.69	10.8
40.	44.35	0.03	1.33	3.54	17.5	2.62	10.1	8.29	13.9	1.75	10.1	5.76	13.9
0.	49.49	0.	0.	3.30	17.6	0.	12.4	5.62	16.8	0.	12.4	3.91	16.8

Table 11 -- Vertical Bending Stresses (in psi) at upstream and downstream faces of Pine Flat Dam

Elevation y (ft)	Case 1			Case 2			Case 3			Case 4		
	r_1	r_{sc}	r_d	r_1	r_{sc}	r_d	r_1	r_{sc}	r_d	r_1	r_{sc}	r_d
308.85	112	-23	114	185	-25	186	75	-23	78	128	-25	131
300	122	-24	124	200	-27	202	81	-24	85	139	-27	142
280	148	-28	151	238	-31	240	99	-28	103	166	-31	169
240	195	-31	197	302	-36	304	130	-31	134	210	-36	213
200	231	-31	233	350	-36	351	154	-31	157	243	-36	246
160	255	-26	257	383	-30	384	171	-26	173	266	-30	268
120	271	-18	271	405	-20	405	181	-18	182	281	-20	282
80	278	-8	278	417	-6	417	186	-8	186	290	-6	290
40	279	5	279	422	12	422	187	5	187	294	12	294
0	276	20	278	422	32	423	185	20	186	293	32	295

FIGURES

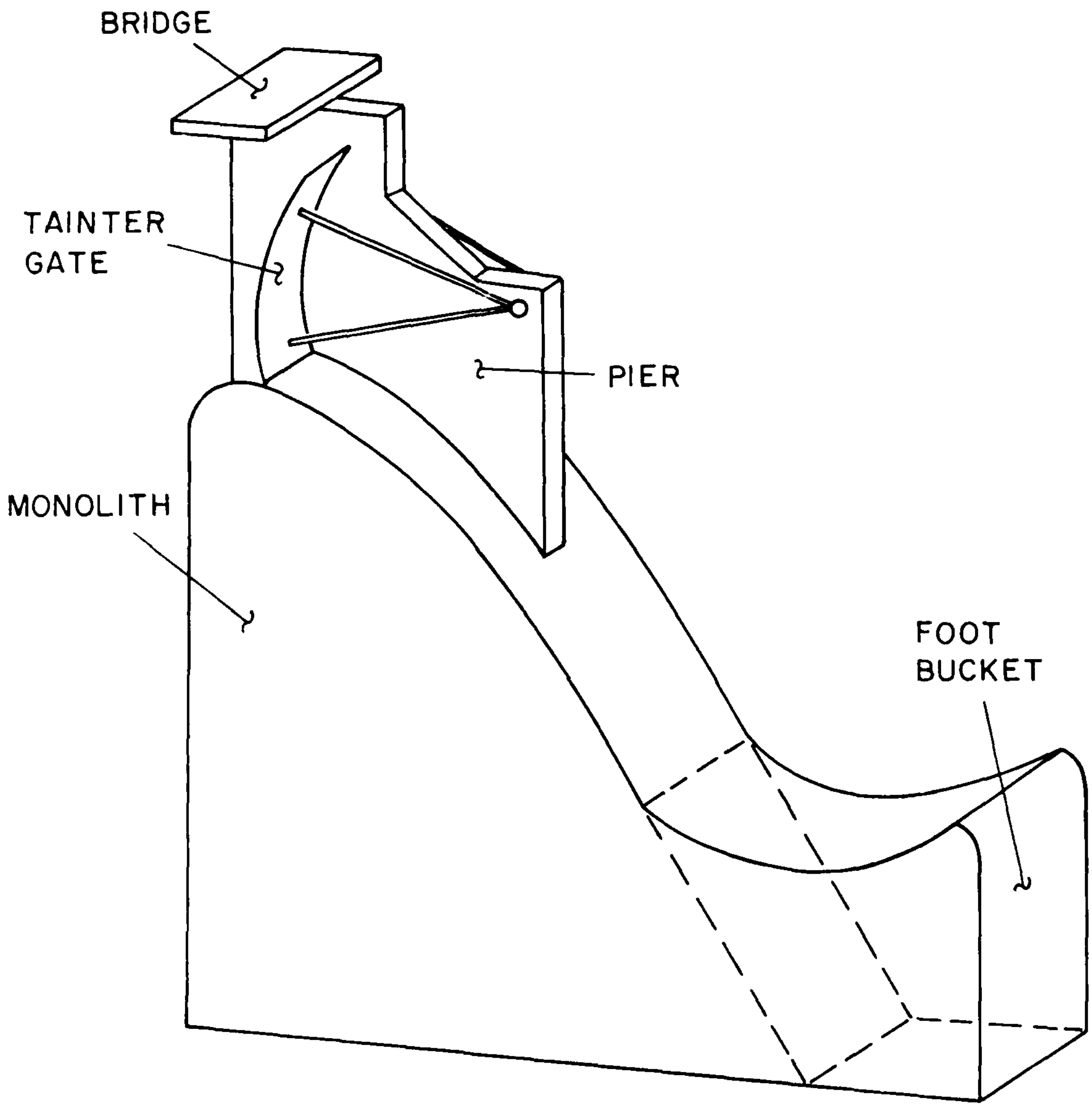

Figure 1 -- Gated Spillway Monolith

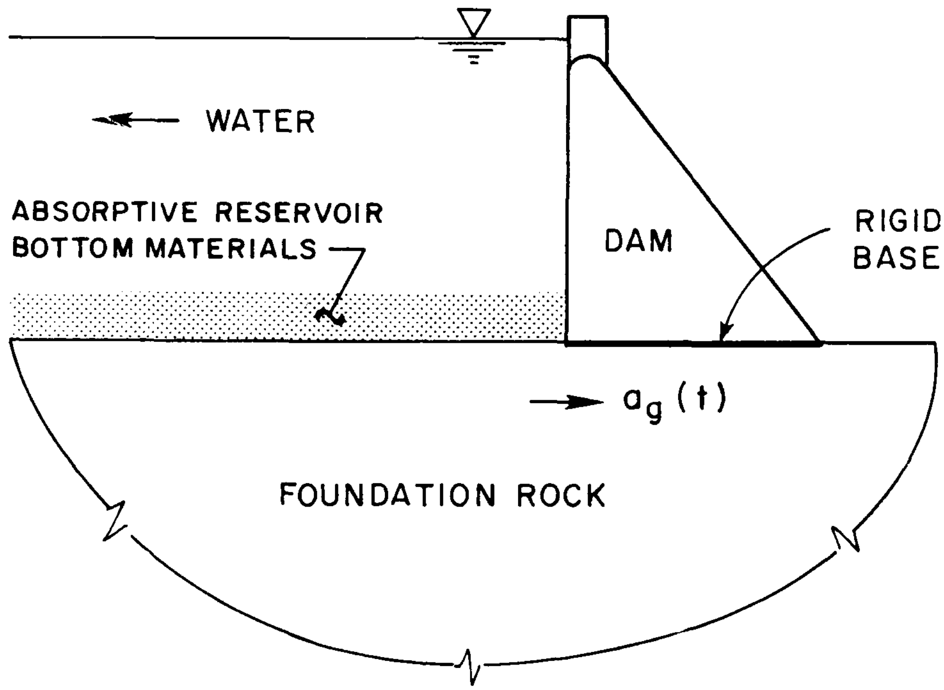

Figure 2 -- Dam-Water-Foundation System

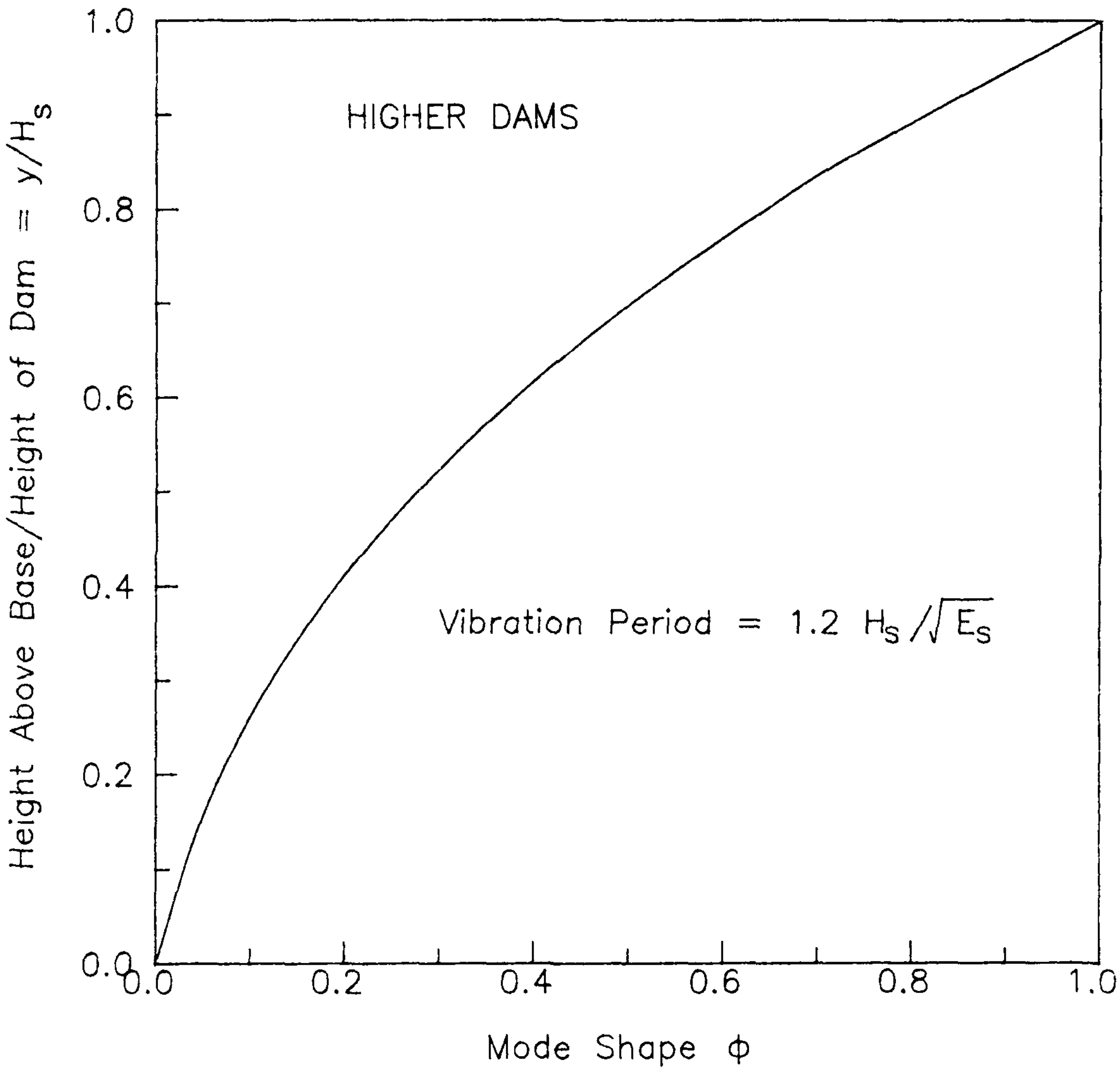

Figure 3(a) -- Standard Period and Mode Shape of Vibration for Gated Spillway Monoliths of Concrete Gravity Dams

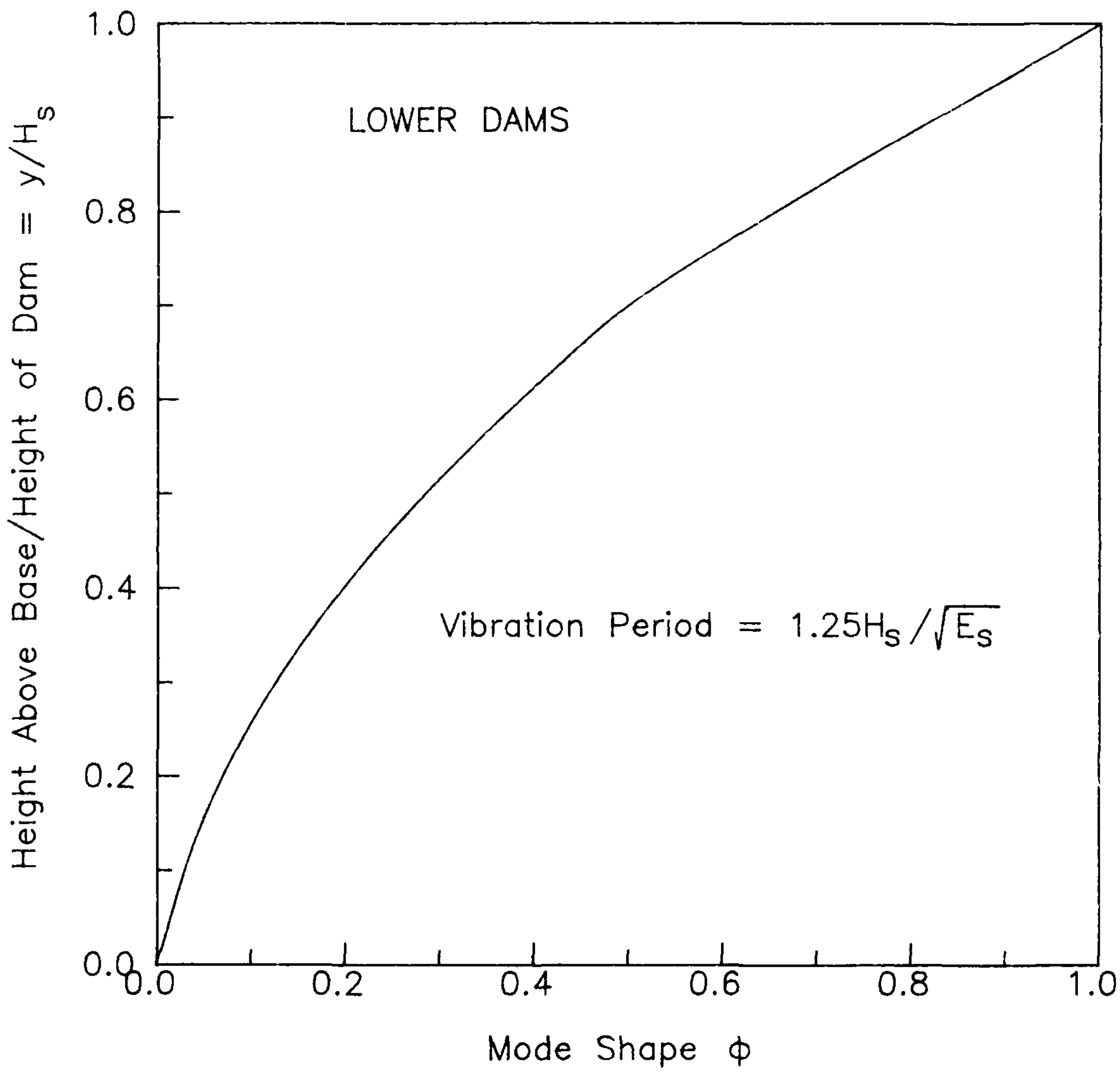

Figure 3(b) -- Standard Period and Mode Shape of Vibration for Gated Spillway Monoliths of Concrete Gravity Dams

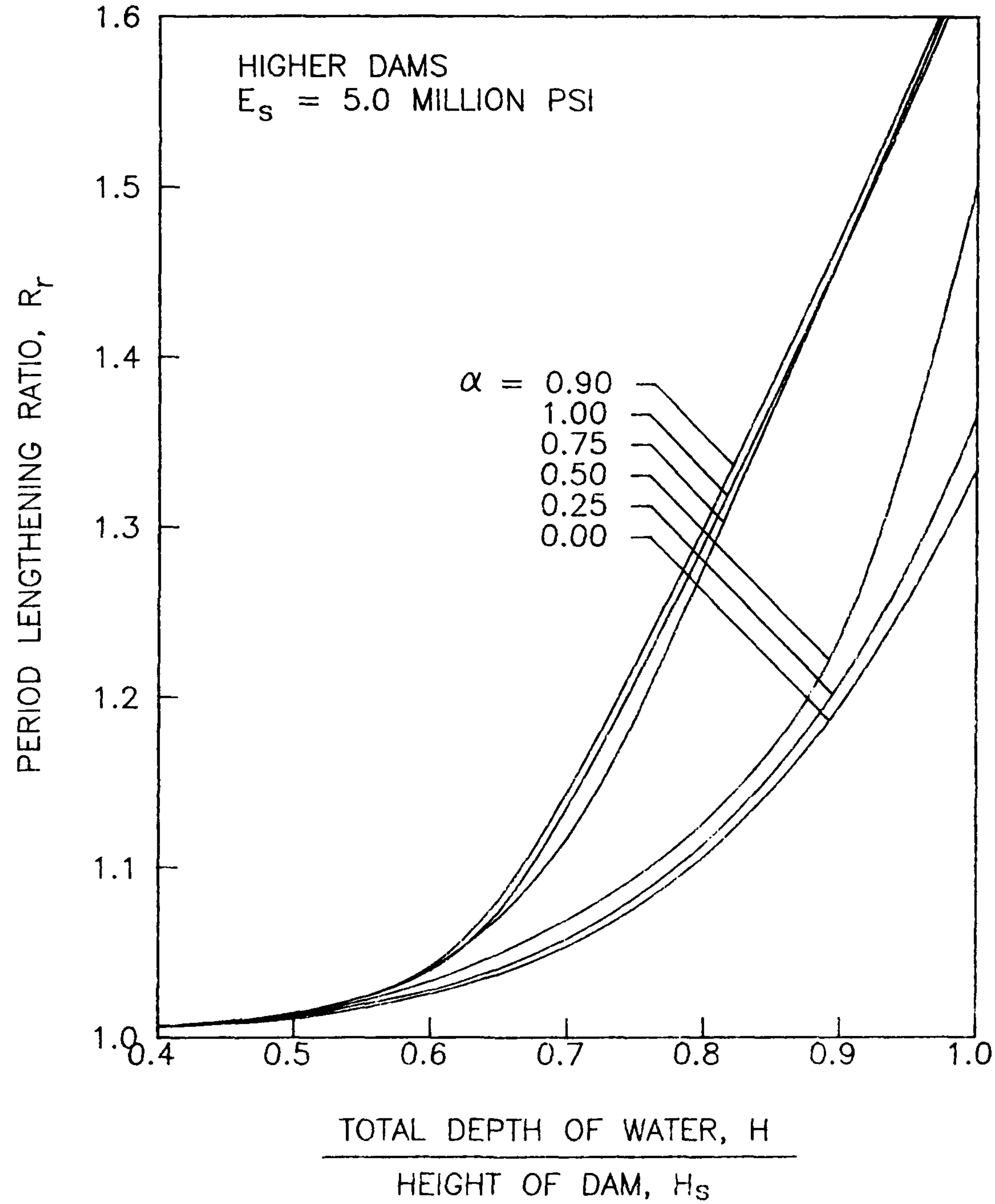

Figure 4(a) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 5 million psi.

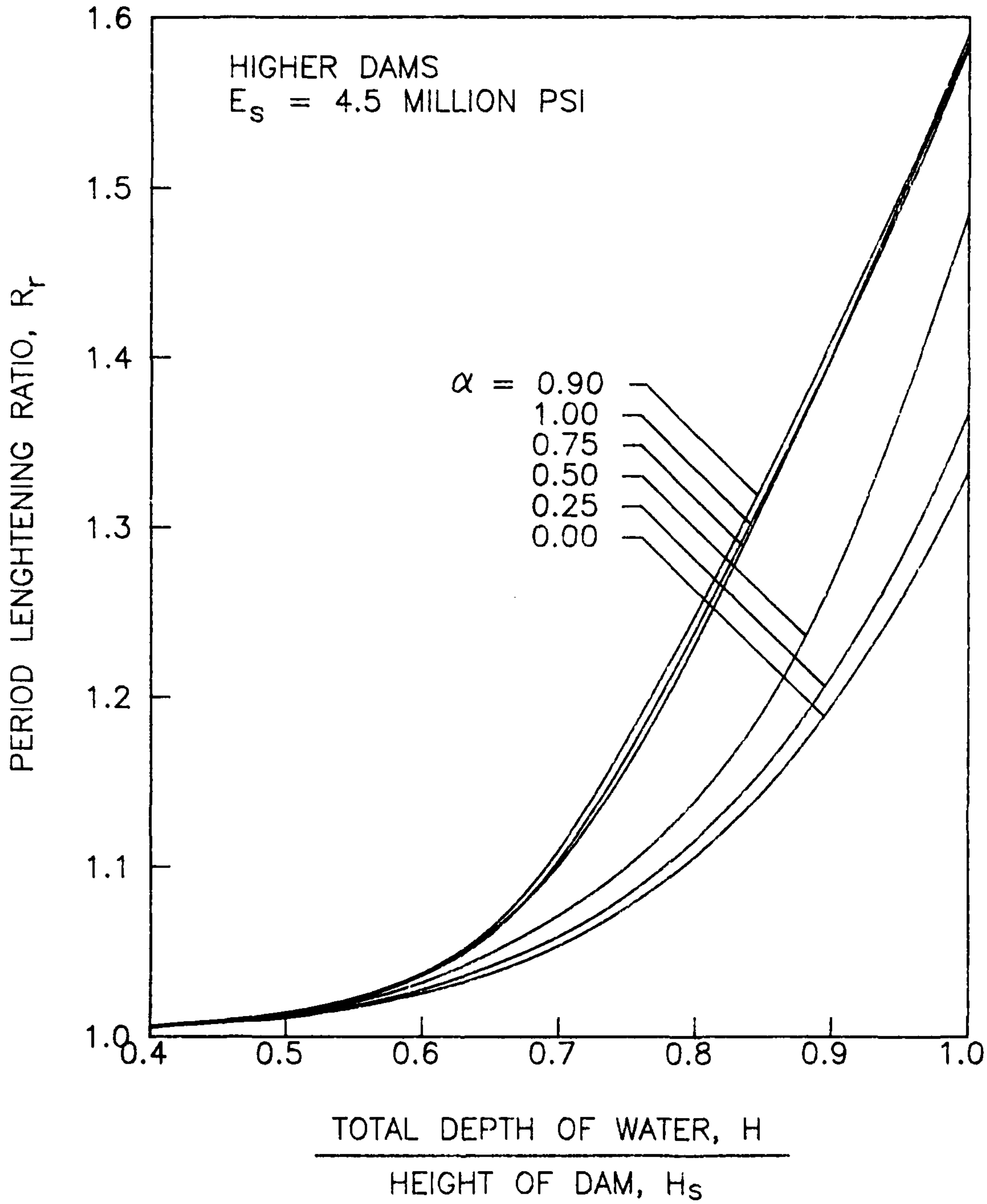

Figure 4(b) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 4.5 million psi.

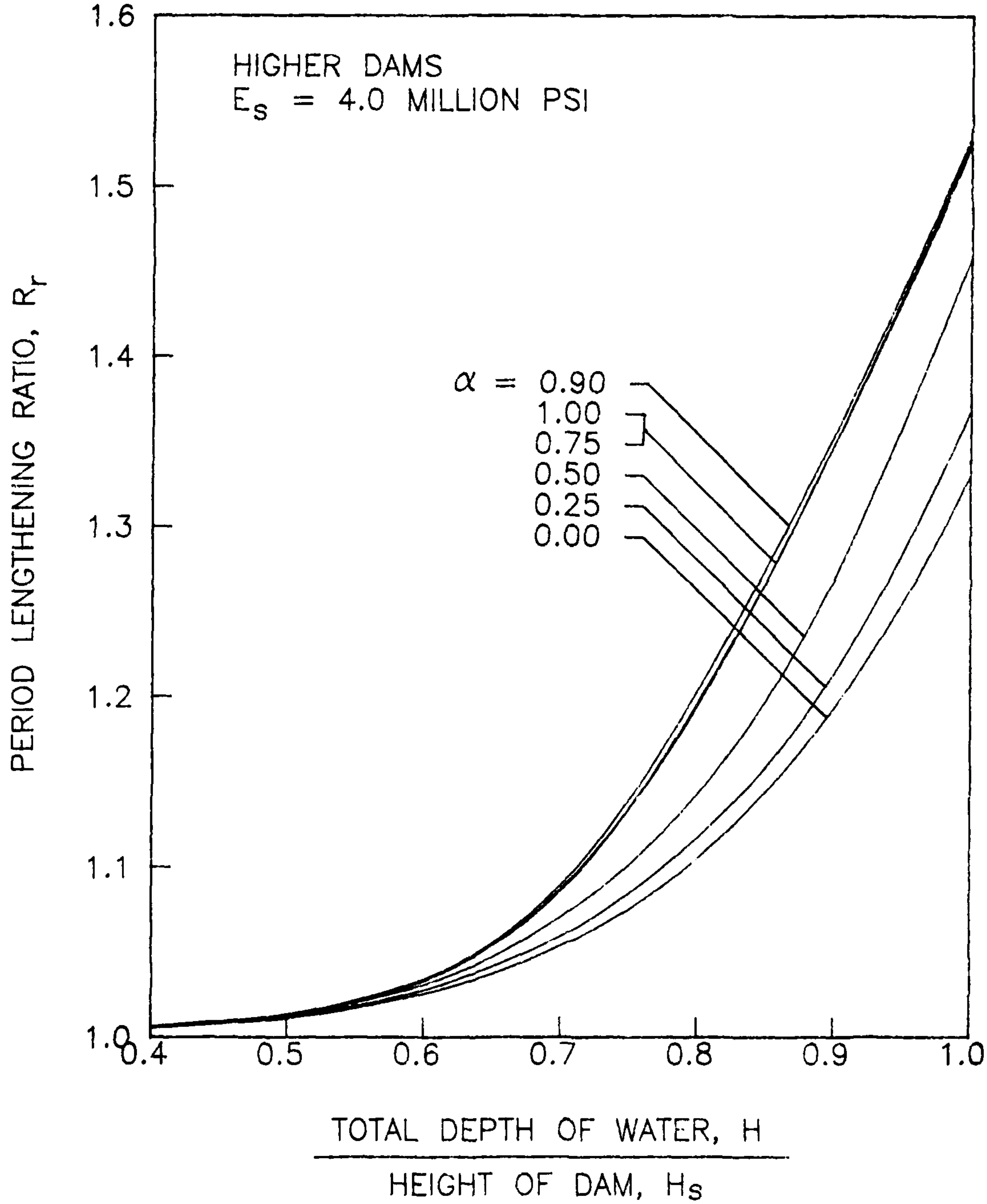

Figure 4(c) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 4 million psi.

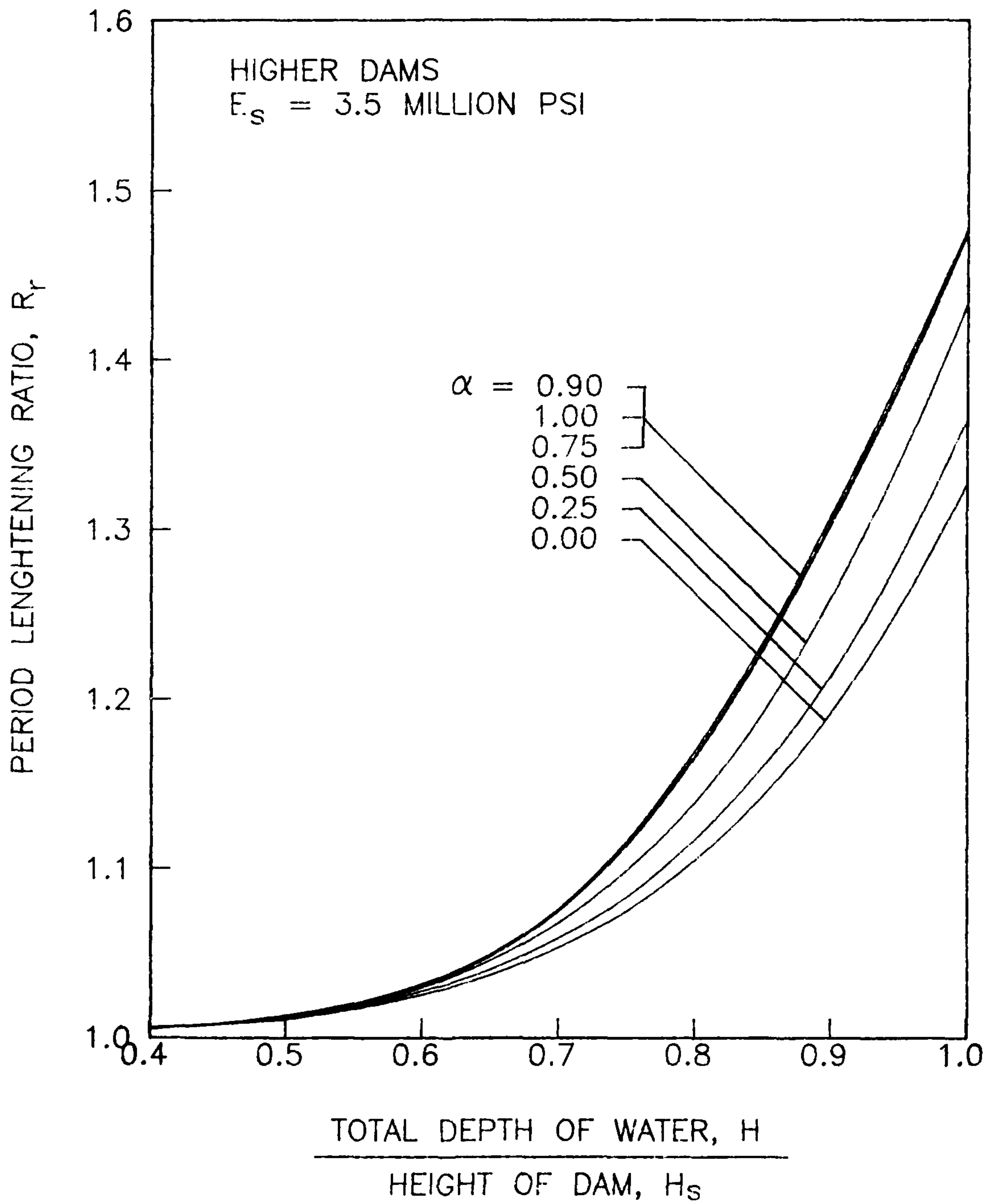

Figure 4(d) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 3.5 million psi.

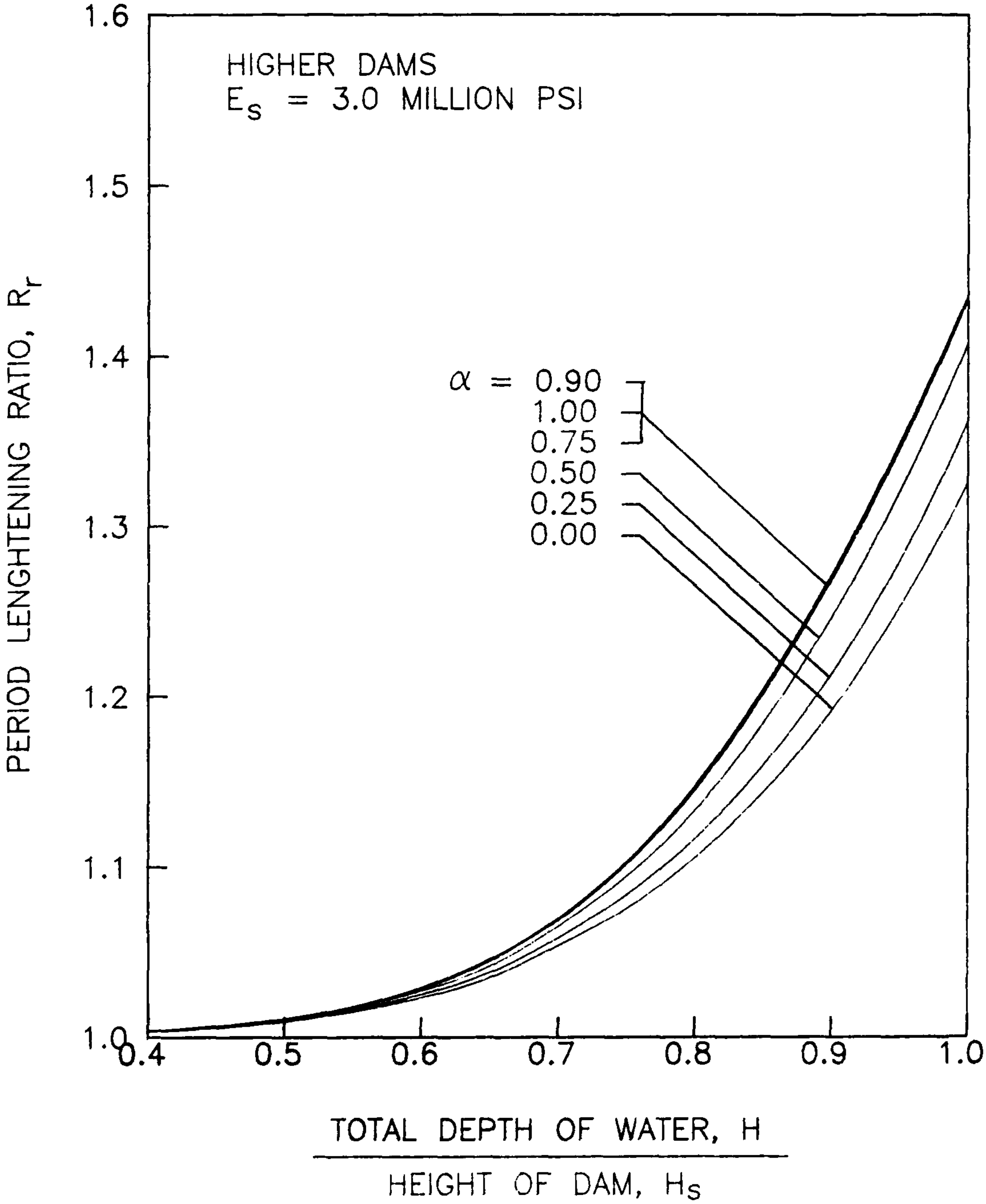

Figure 4(e) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_S = 3 million psi.

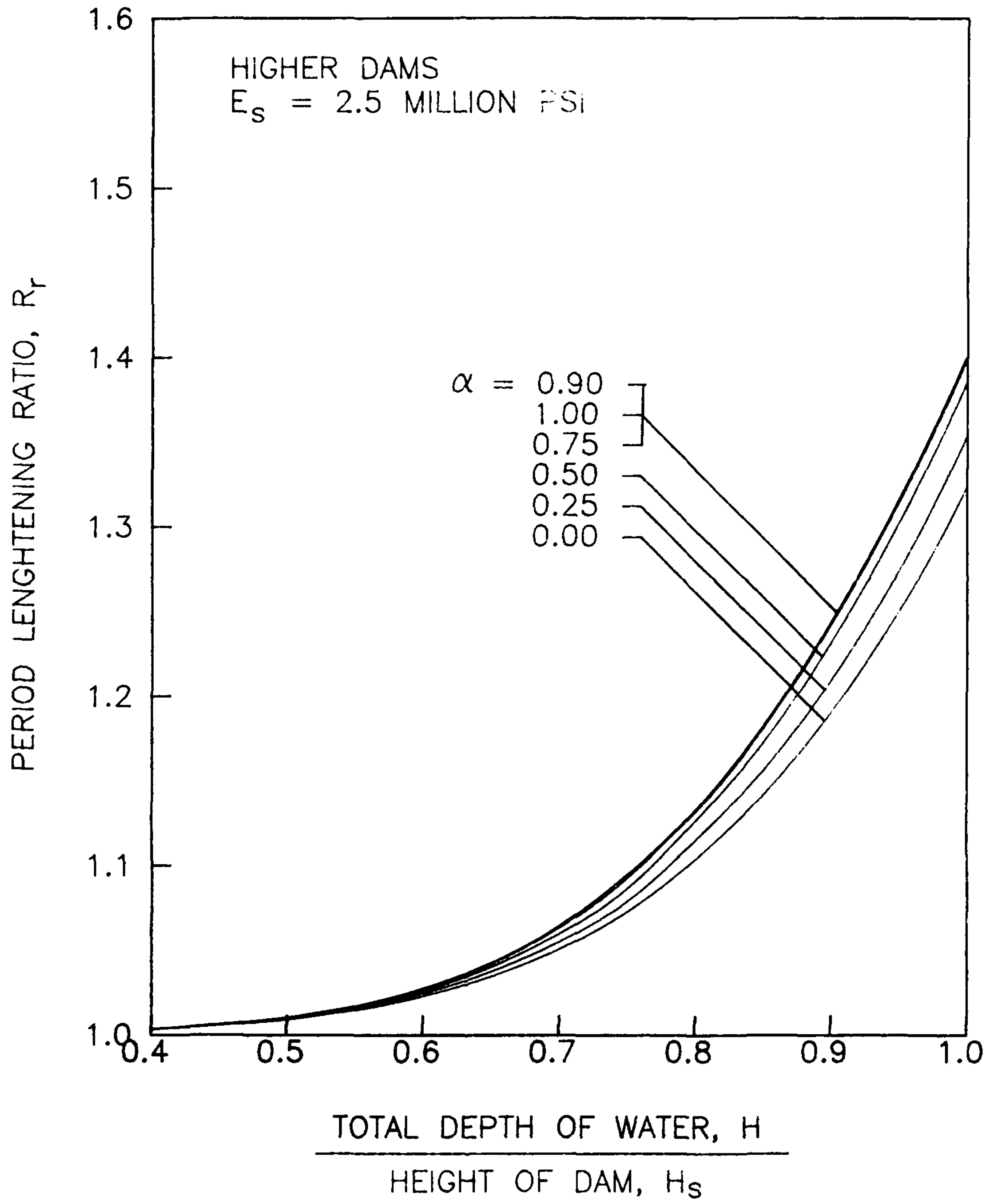

Figure 4(f) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 2.5 miilion psi.

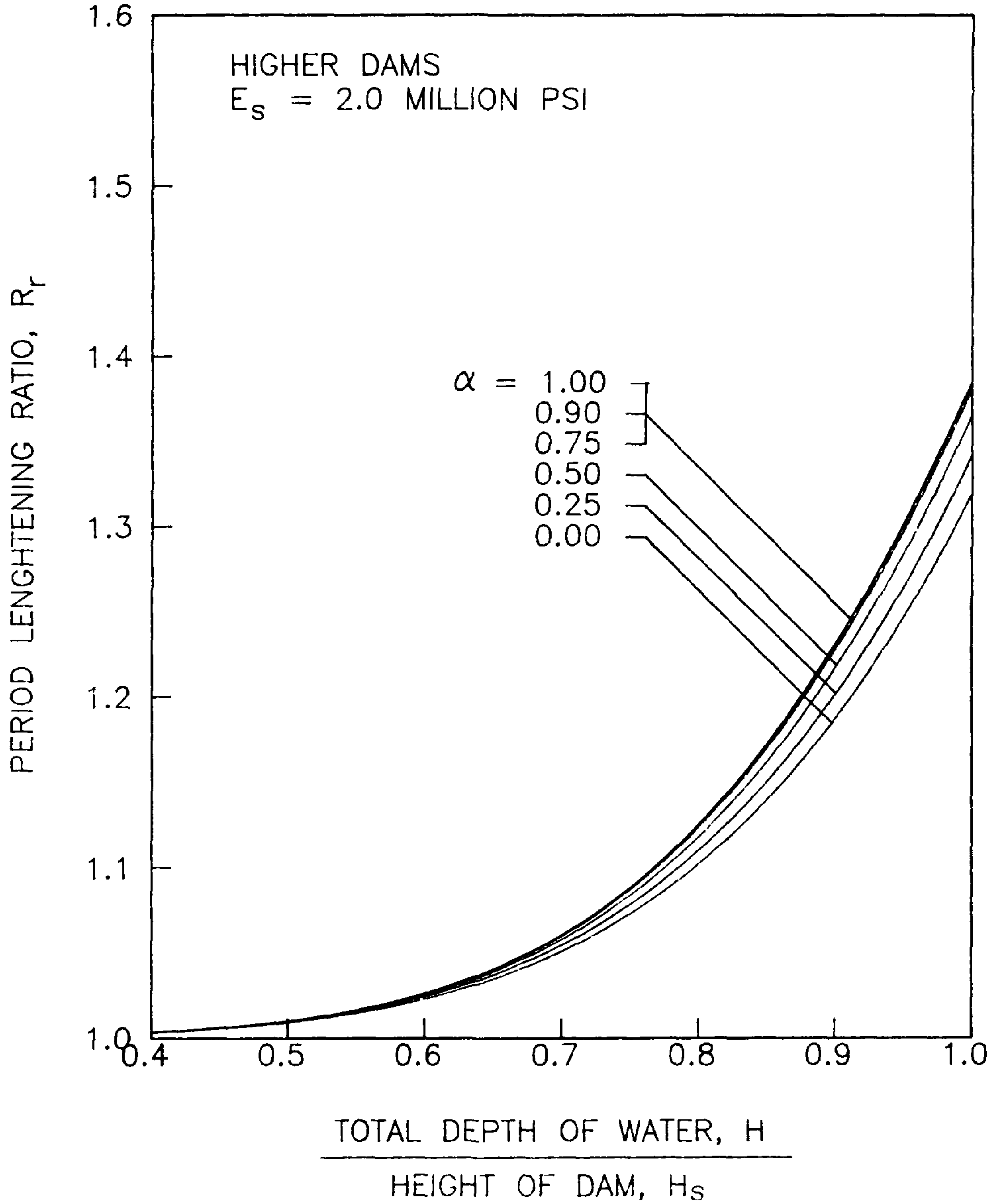

Figure 4(g) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 2 million psi.

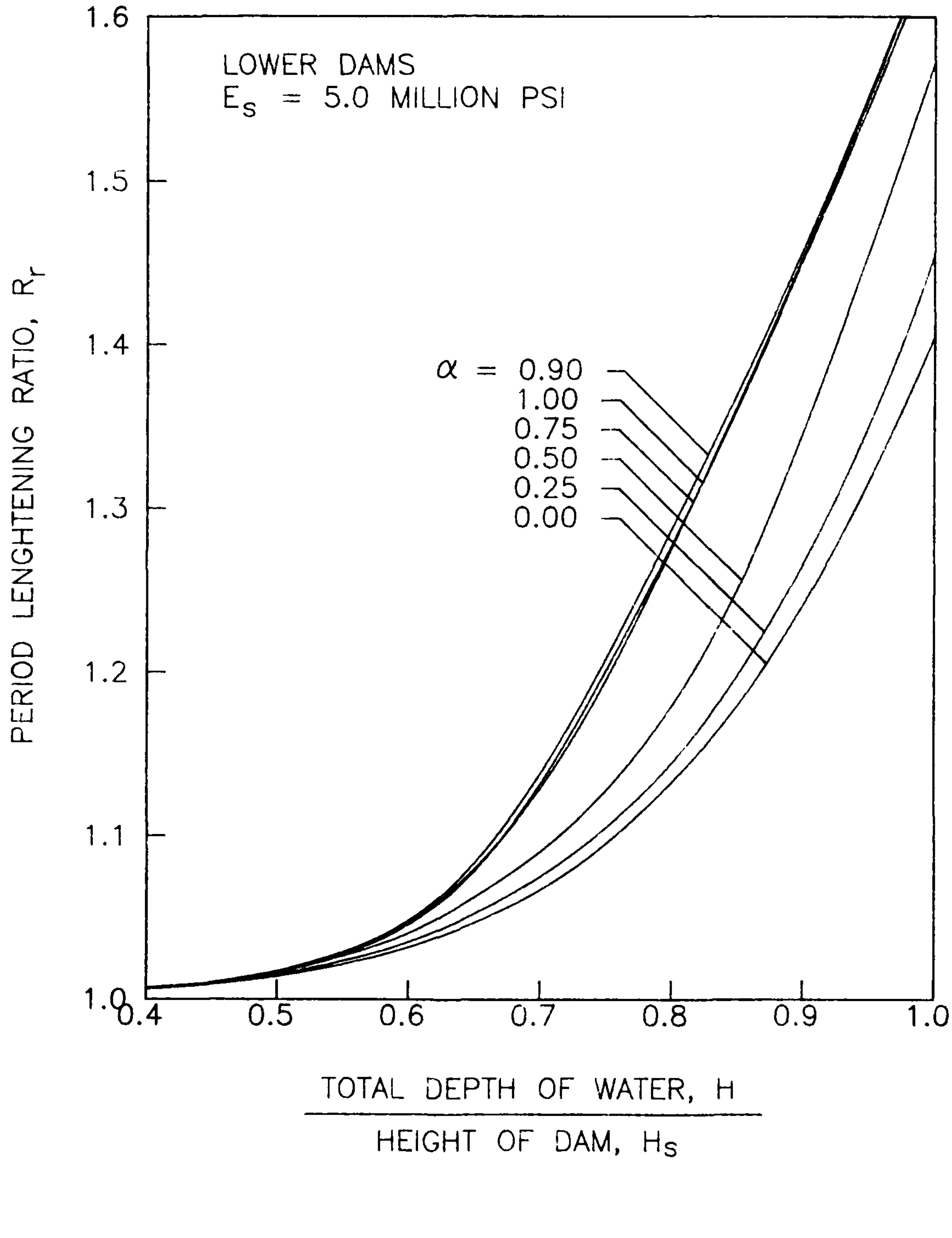

Figure 5(a) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 5 million psi.

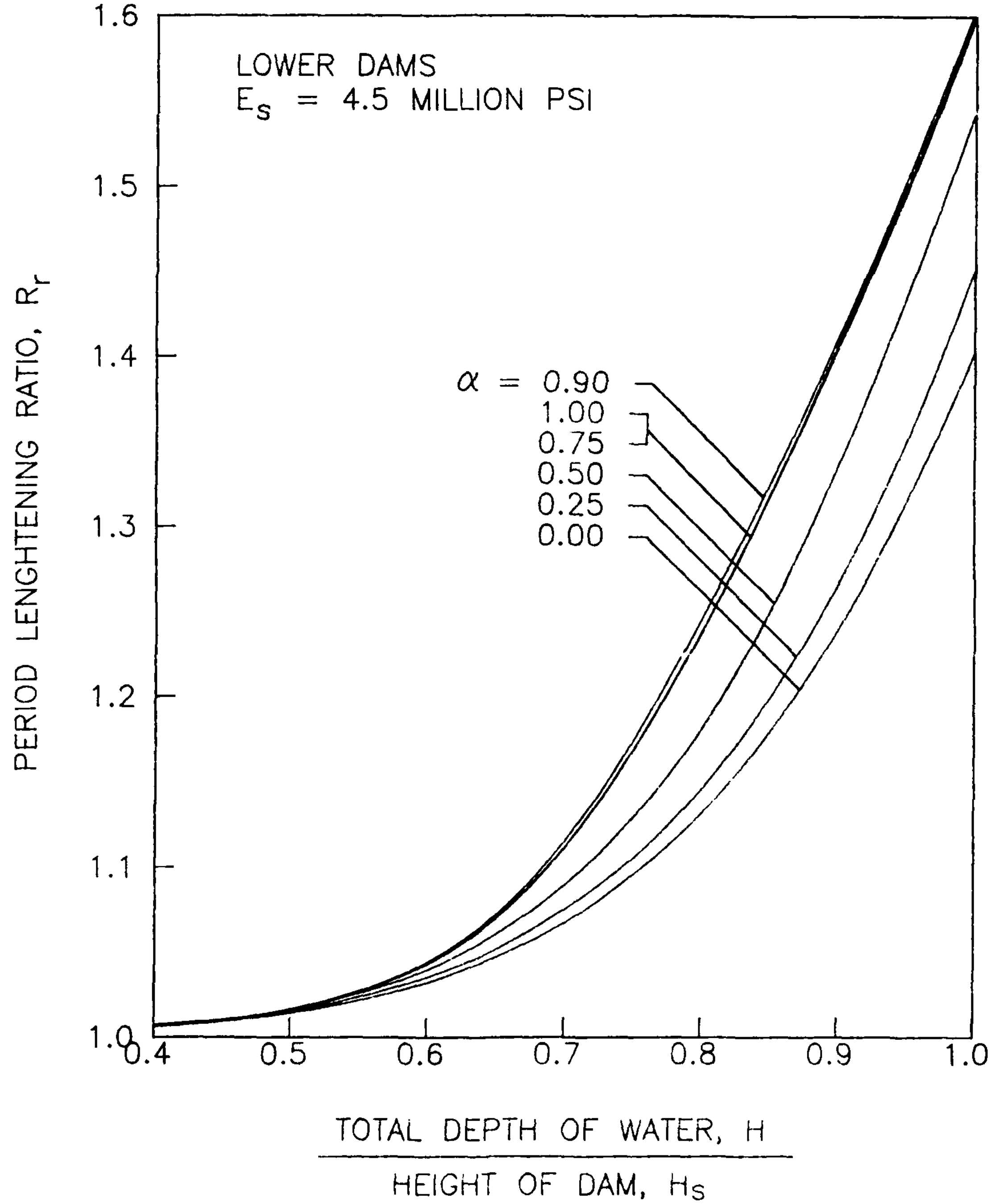

Figure 5(b) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 4.5 million psi.

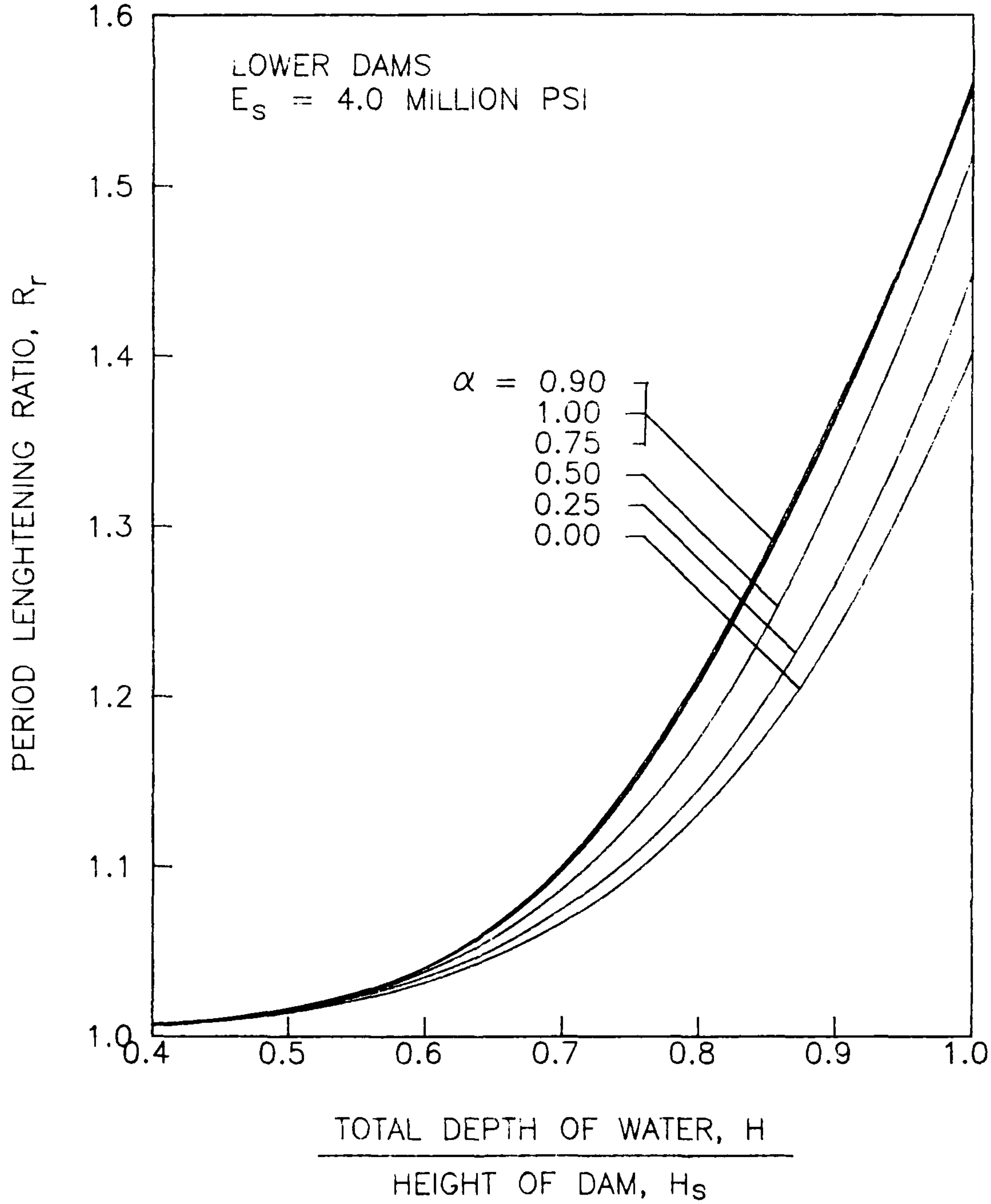

Figure 5(c) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 4 million psi.

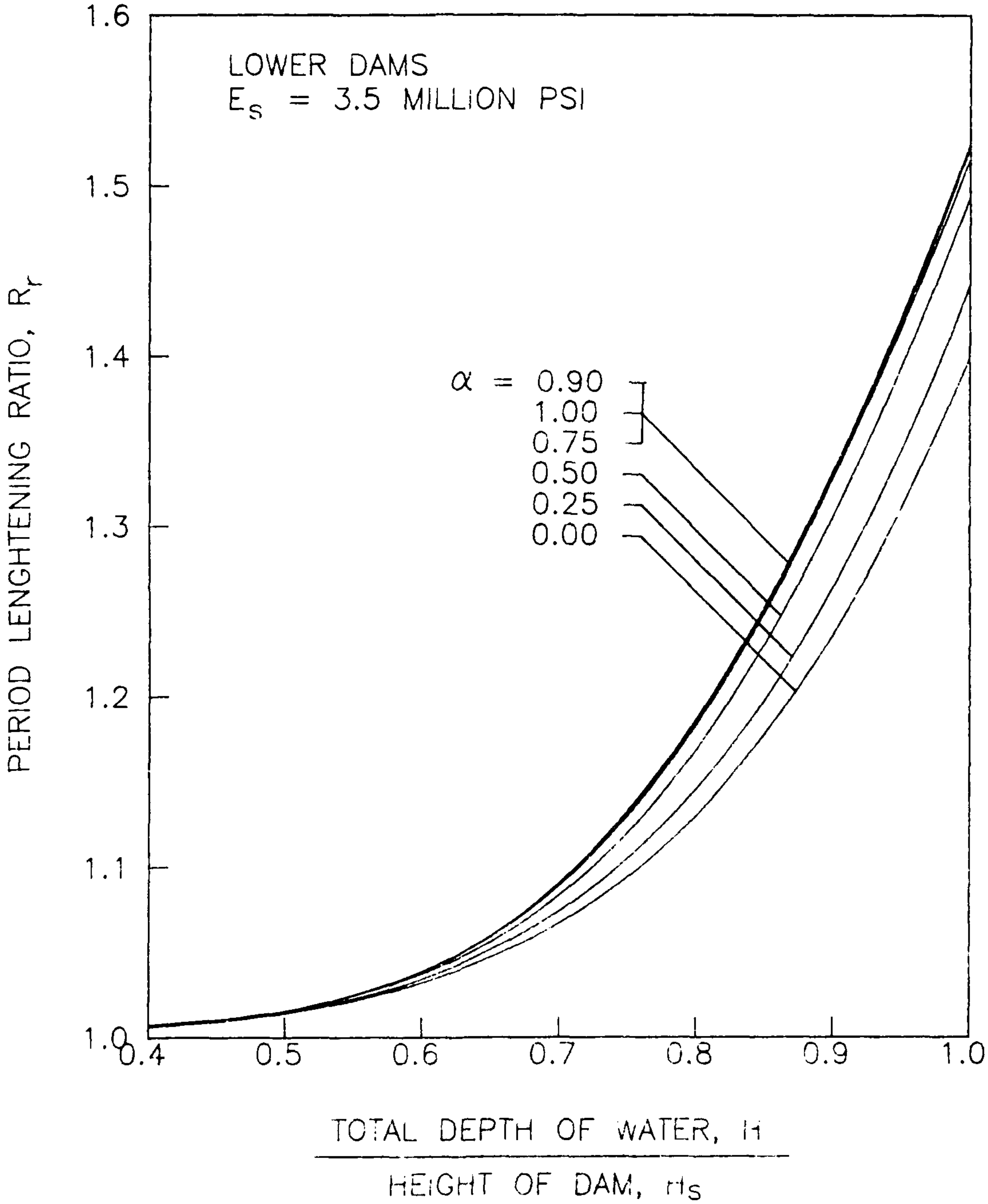

Figure 5(a) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 3.5 million psi.

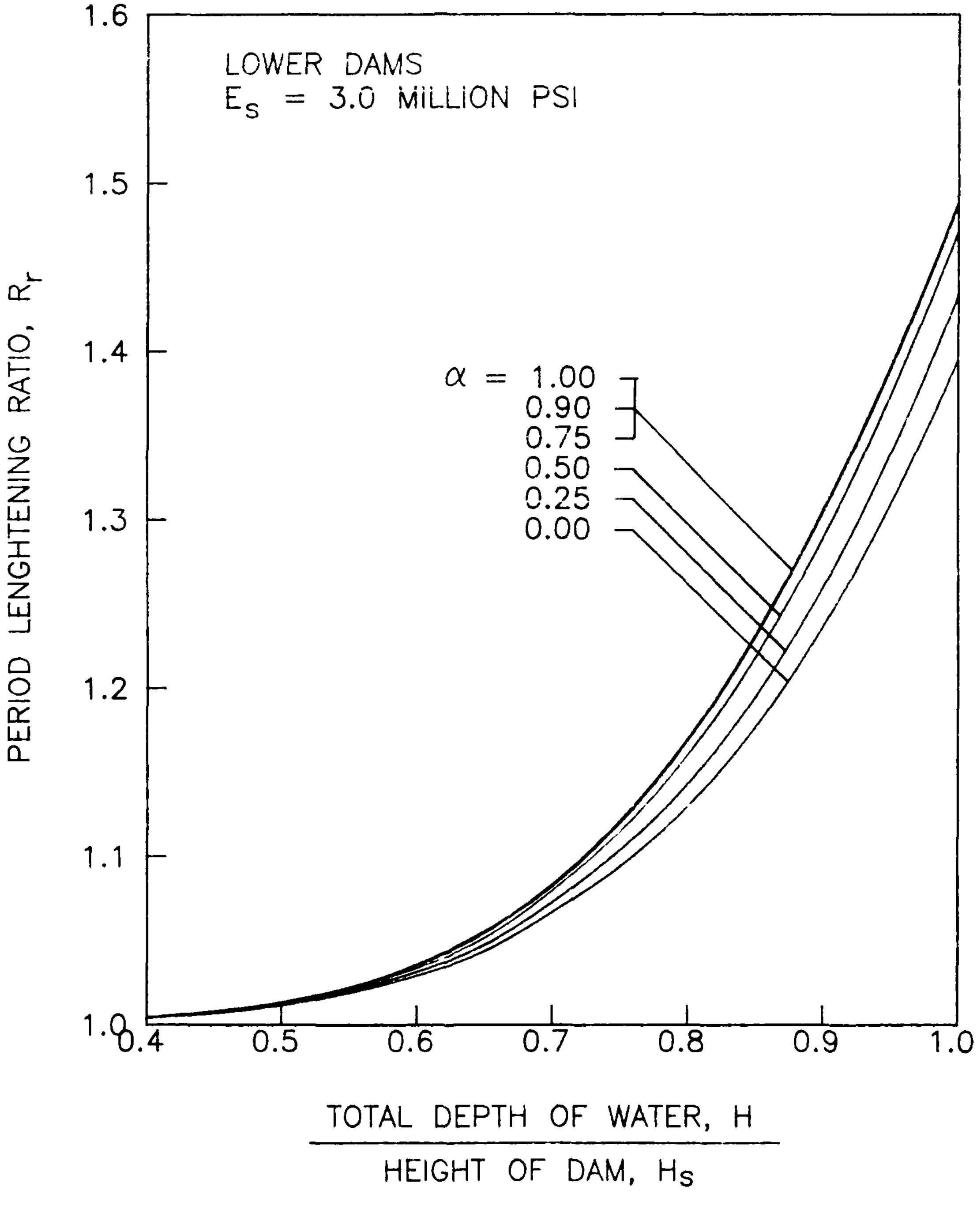

Figure 5(e) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_S = 3 million psi.

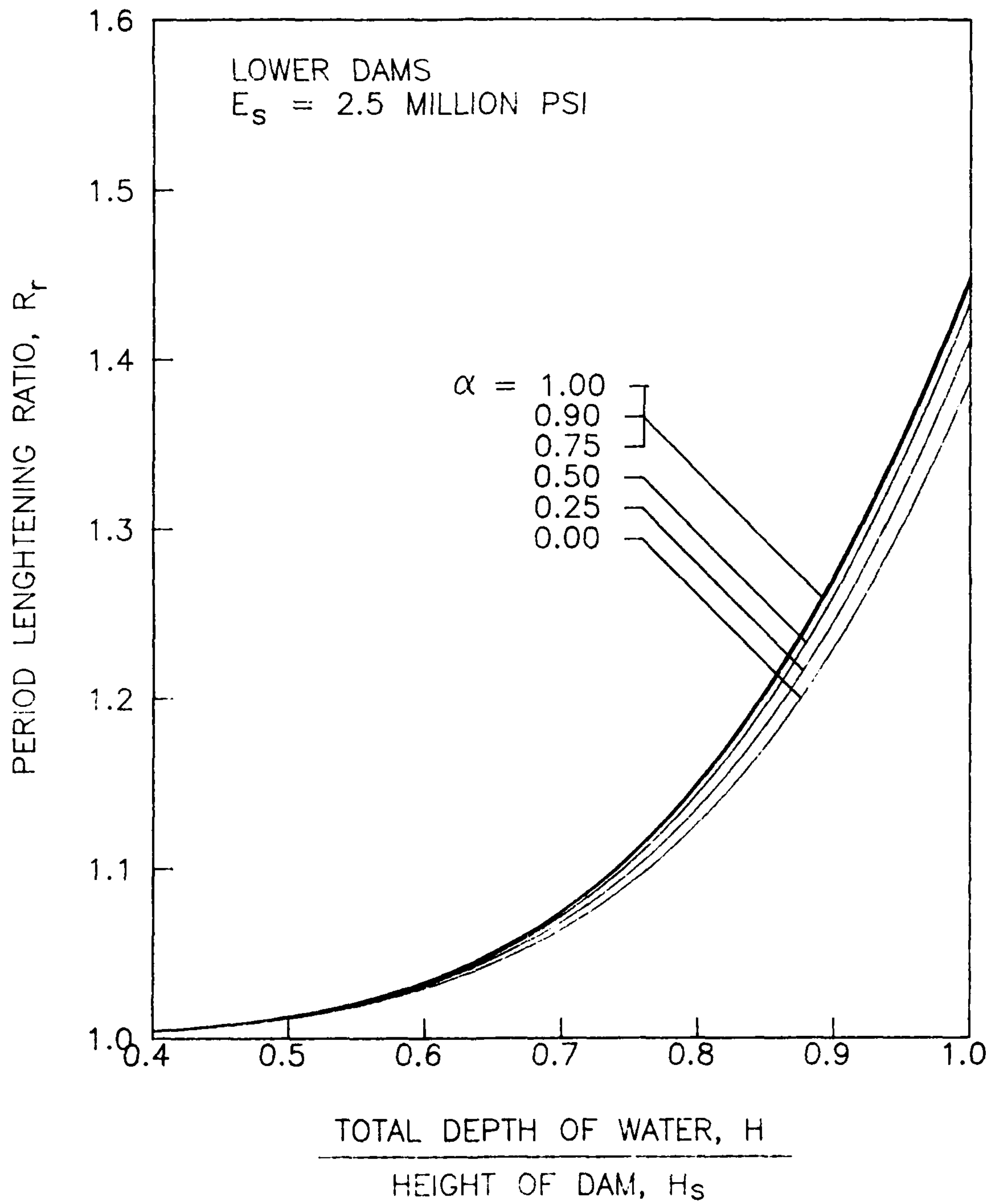

Figure 5(f) —— Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 2.5 million psi.

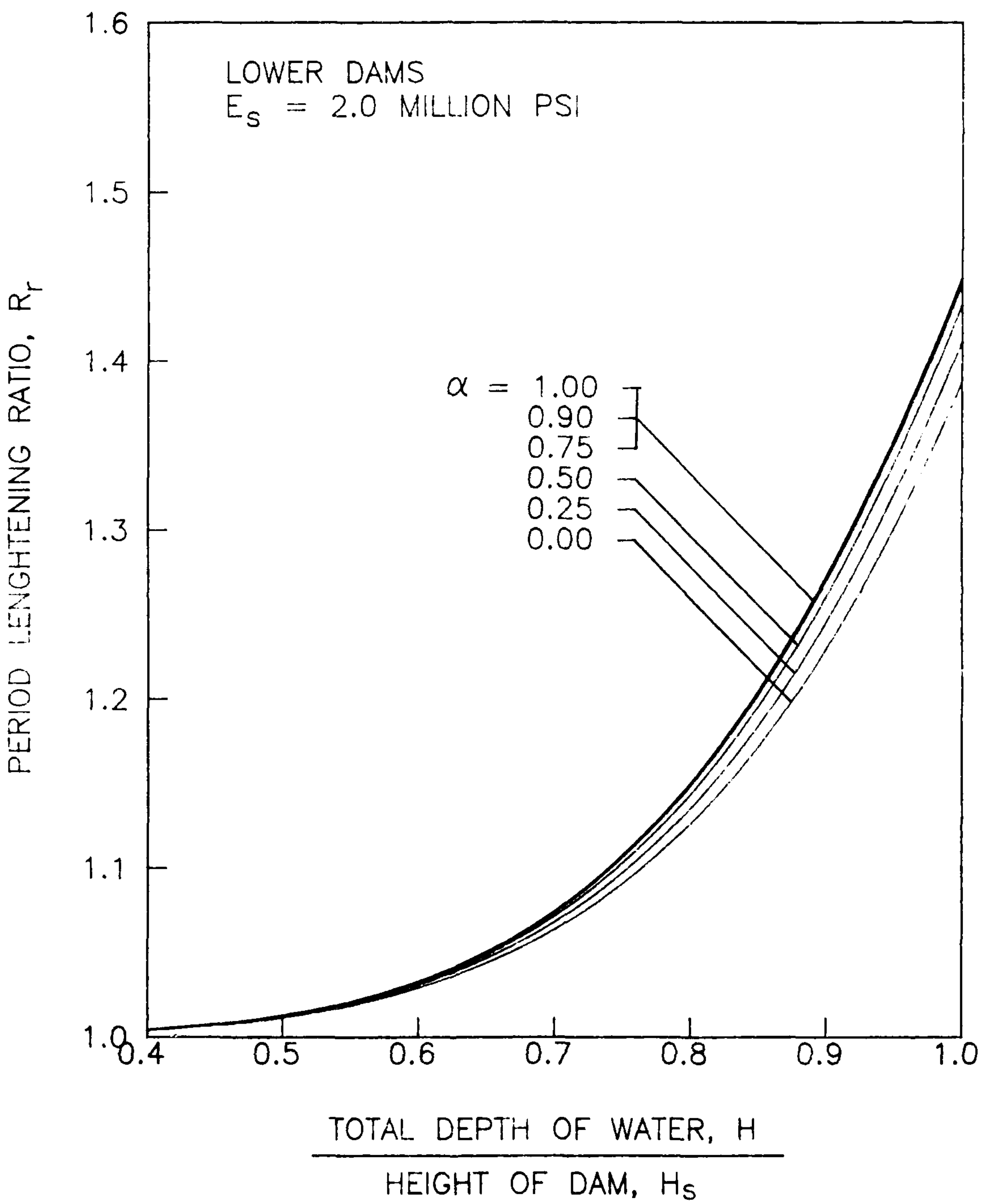

Figure 5(g) -- Standard Values for R_r, the Period Lengthening Ratio due to Hydrodynamic Effects; E_s = 2 million psi.

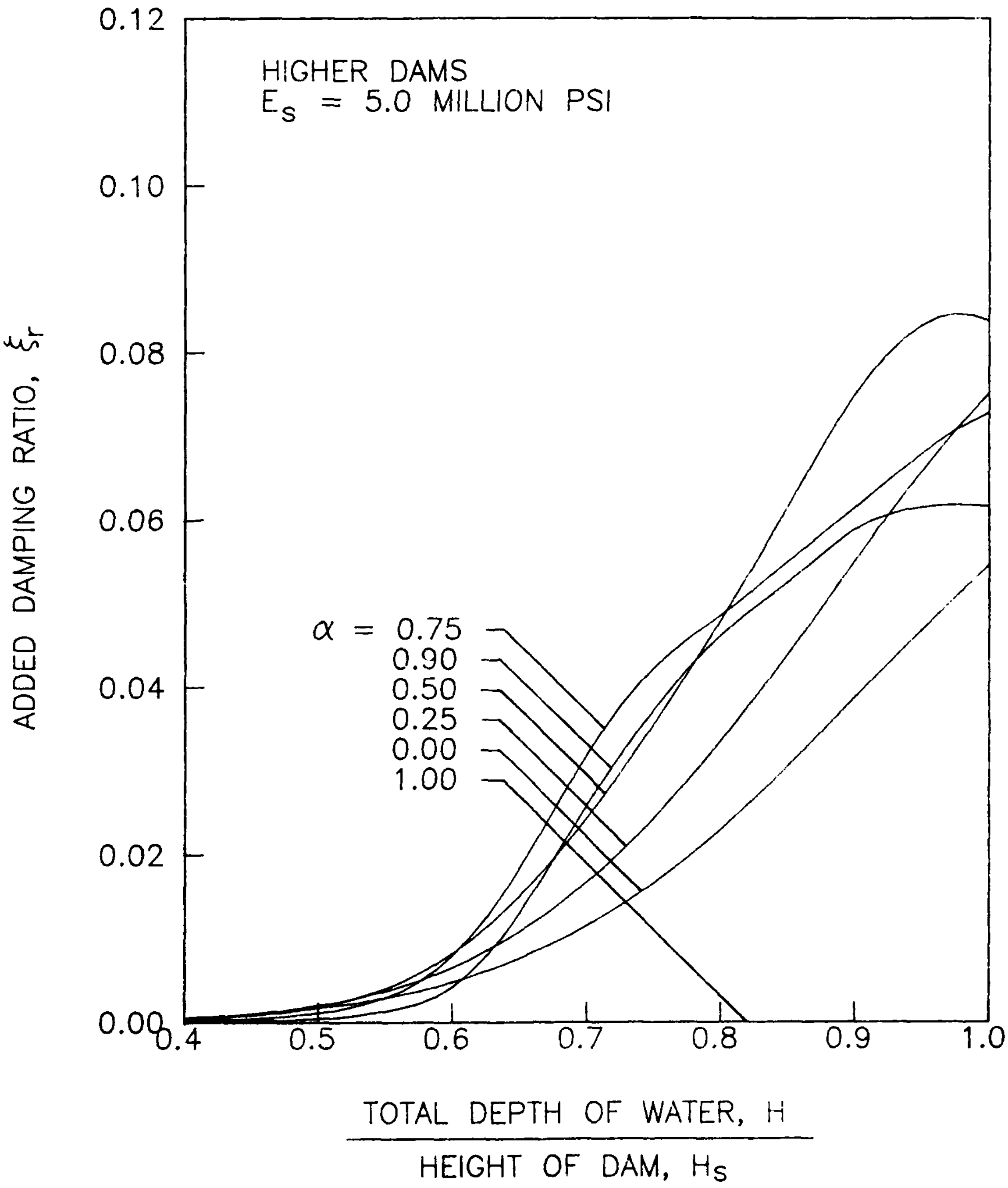

Figure 6(a) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 5 million psi.

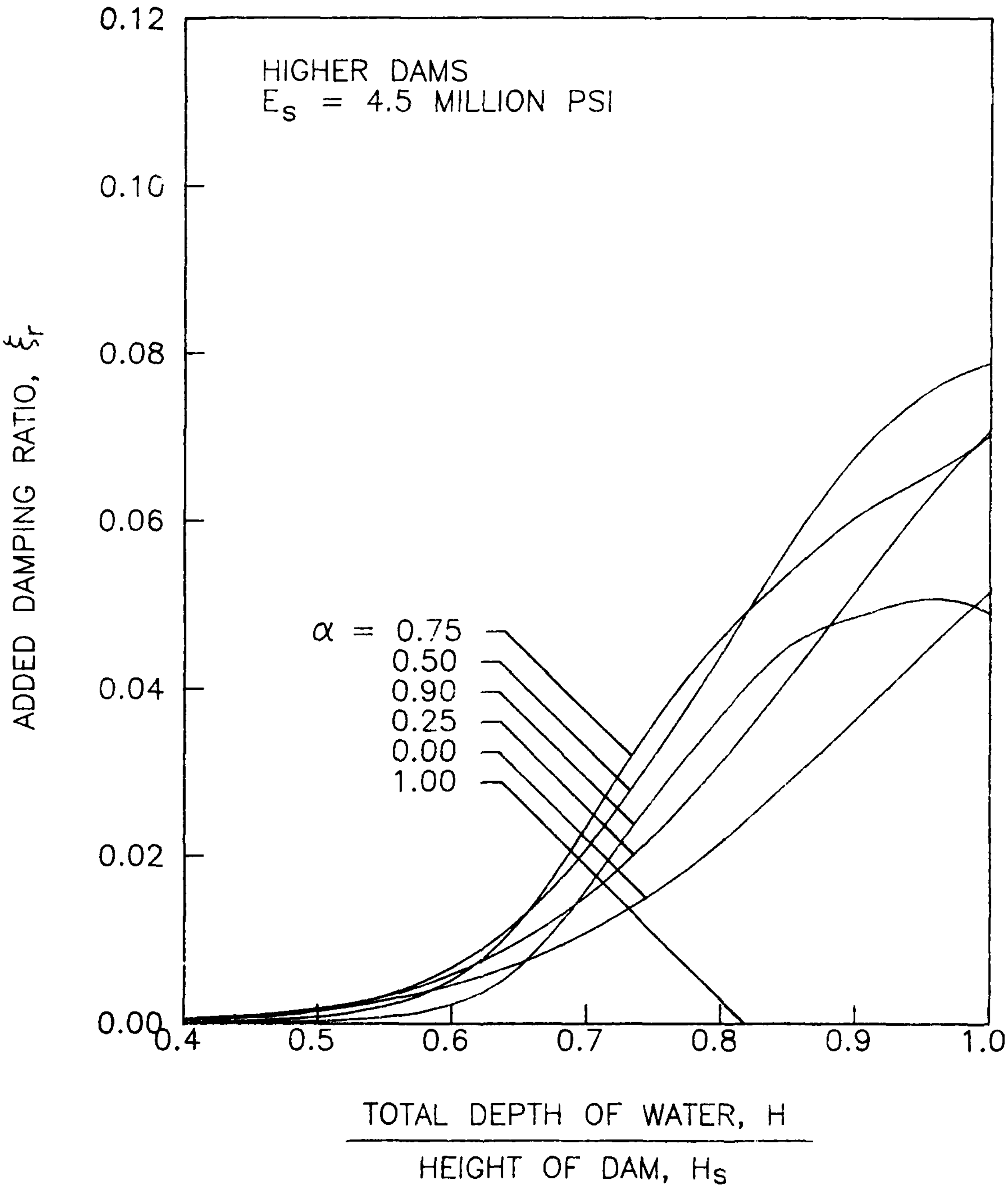

Figure 6(b) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 4.5 million psi.

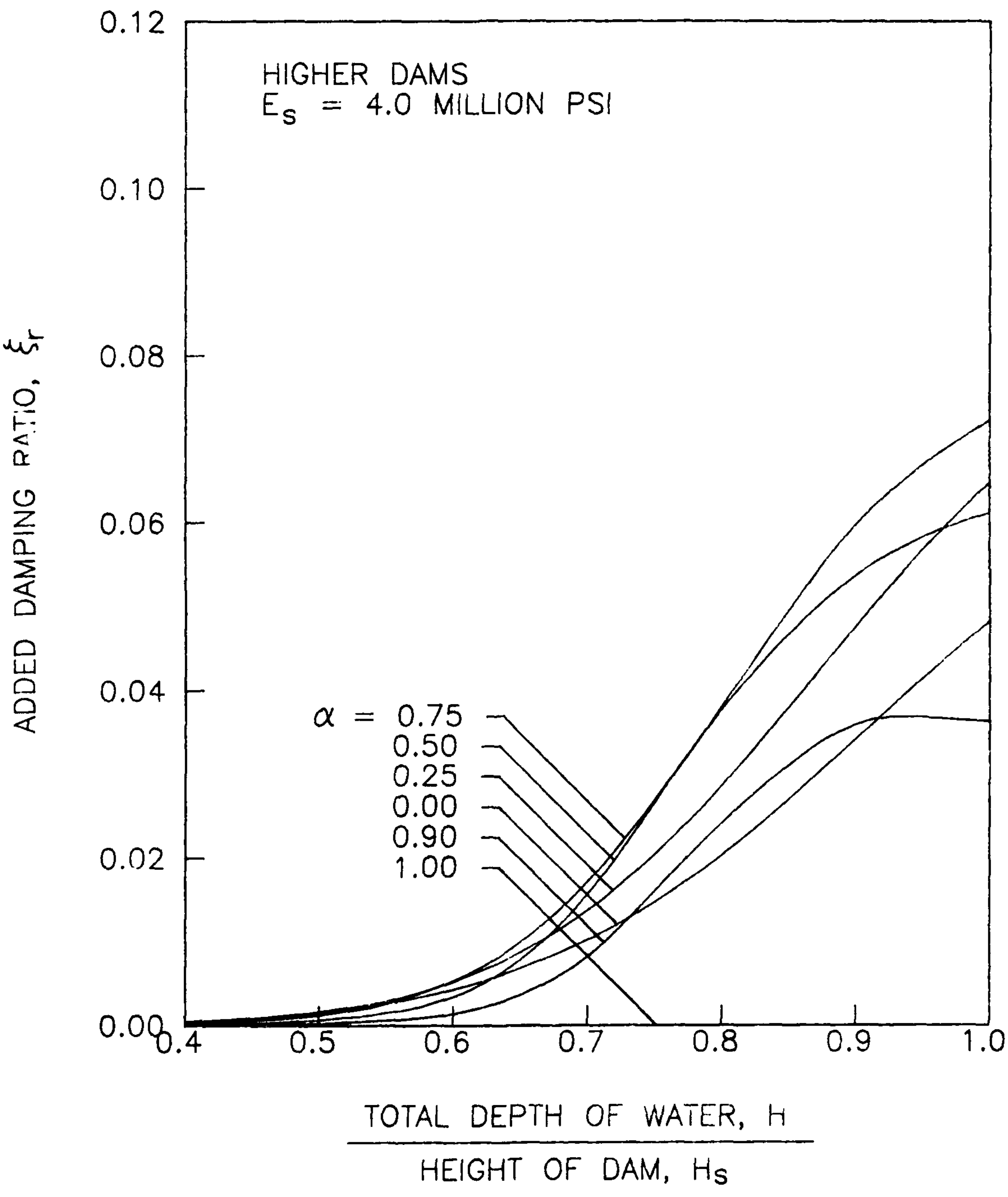

Figure 6(c) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 4 million psi.

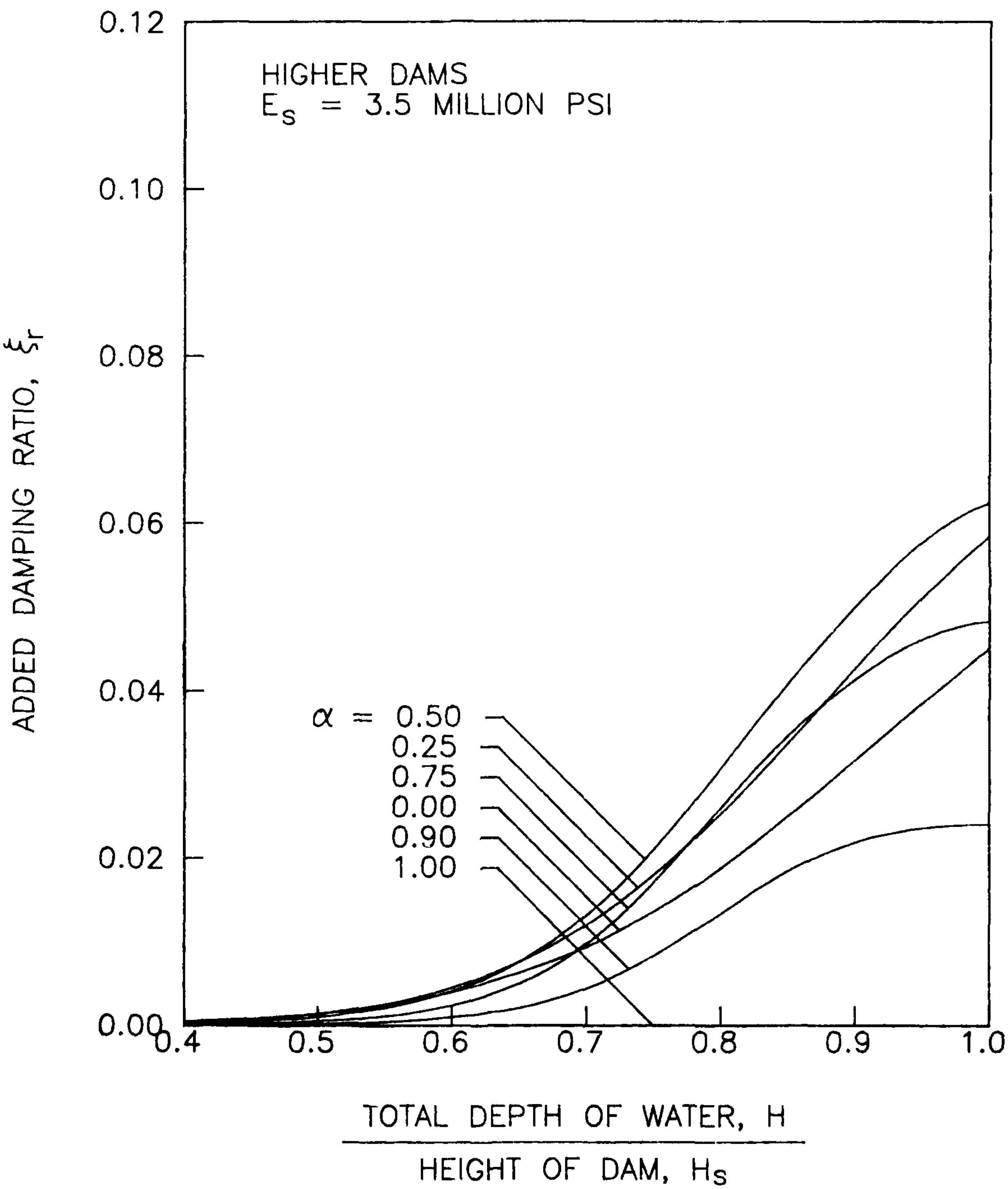

Figure 6(d) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_S = 3.5 million psi.

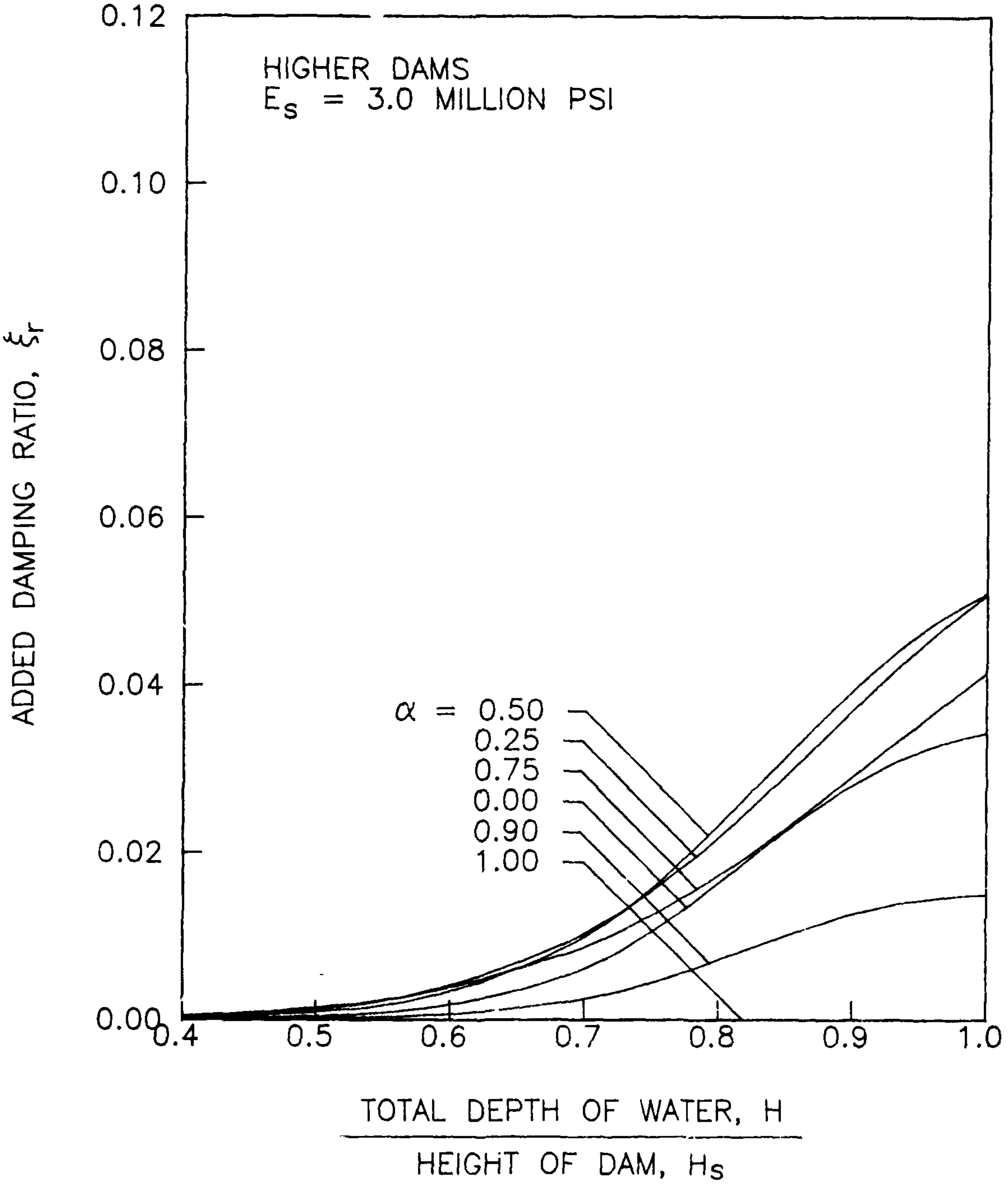

Figure 6(e) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 3 million psi.

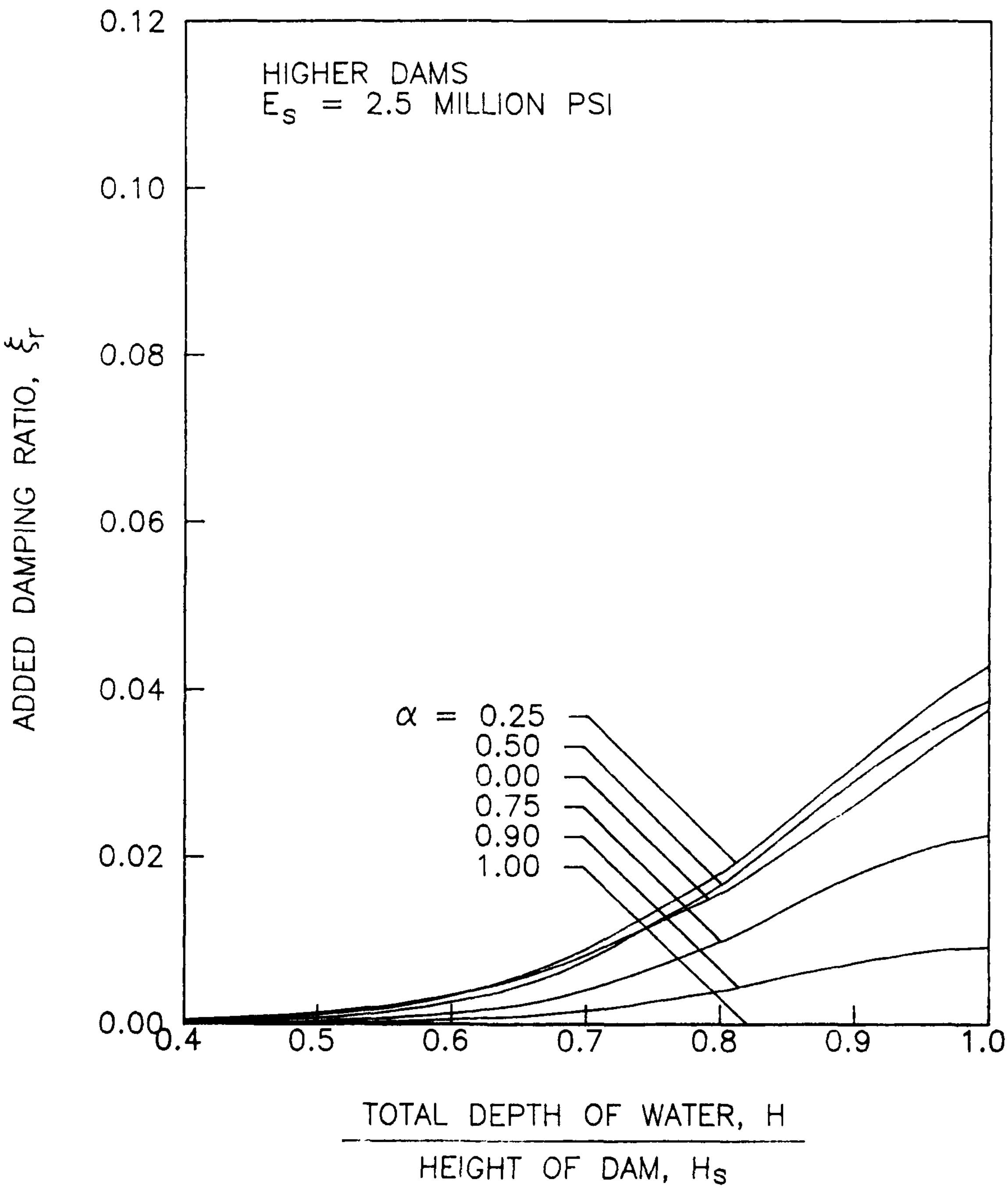

Figure 6(f) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 2.5 million psi.

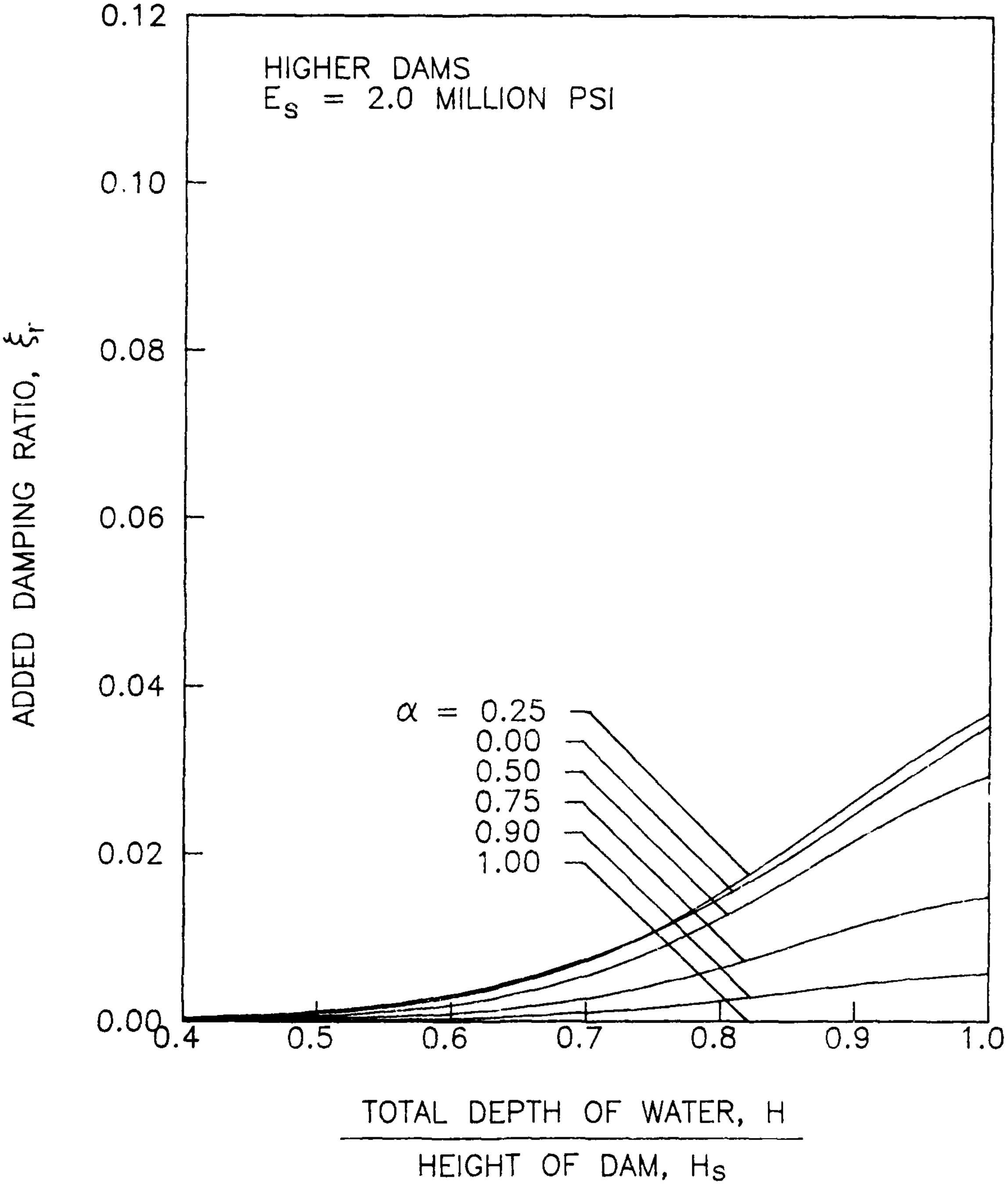

Figure 6(g) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 2 million psi.

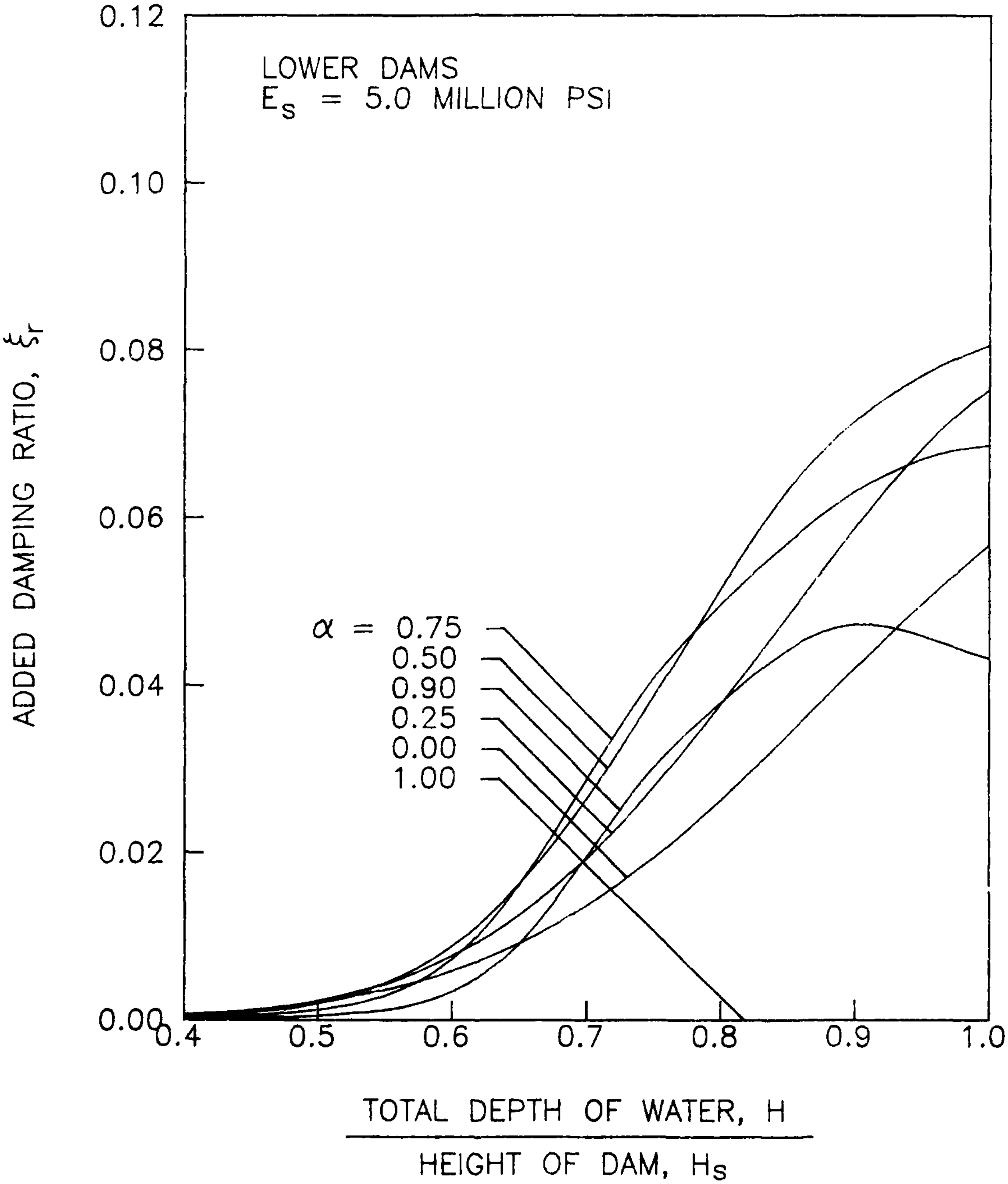

Figure 7(a) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 5 million psi.

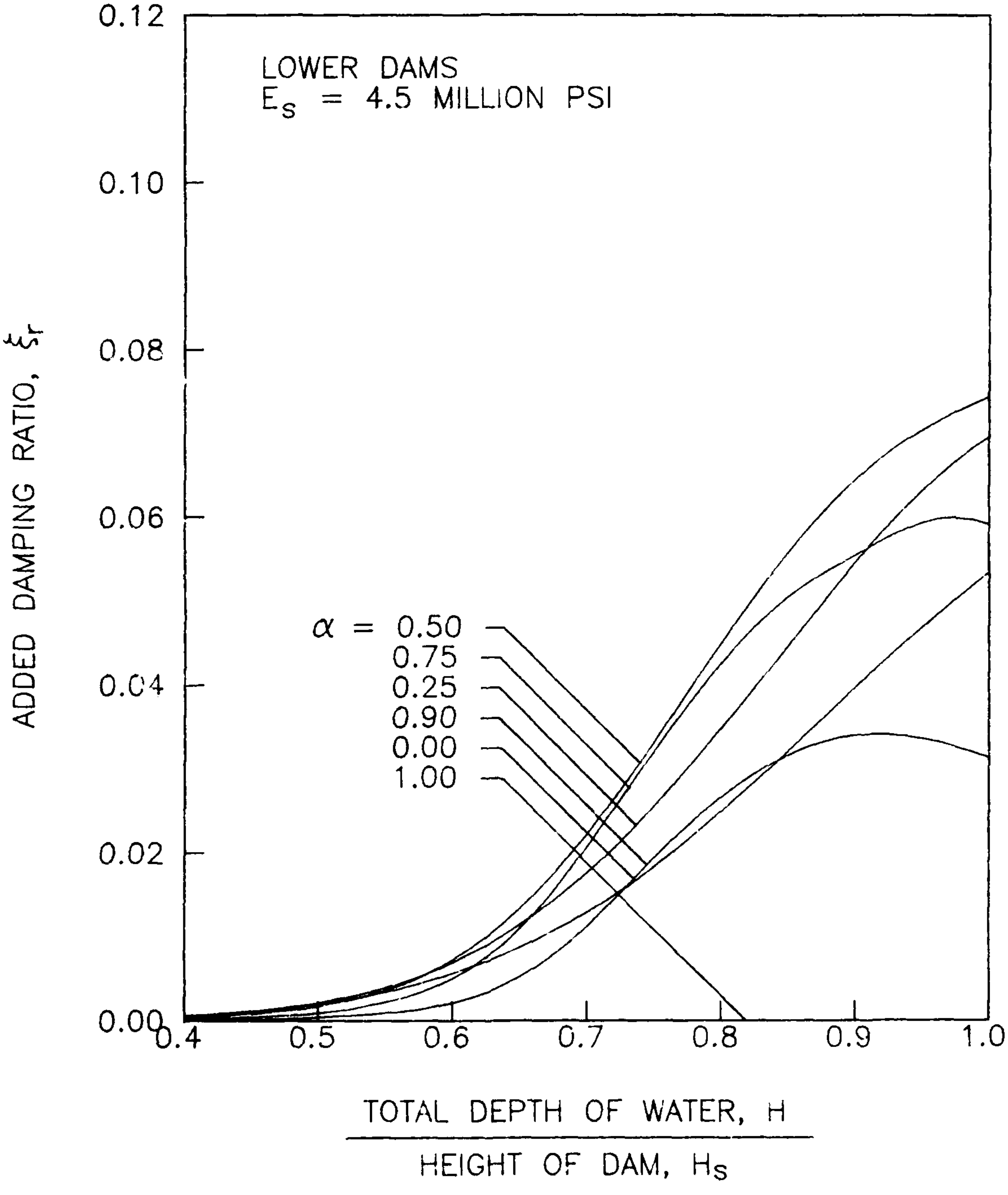

Figure 7(b) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 4.5 million psi.

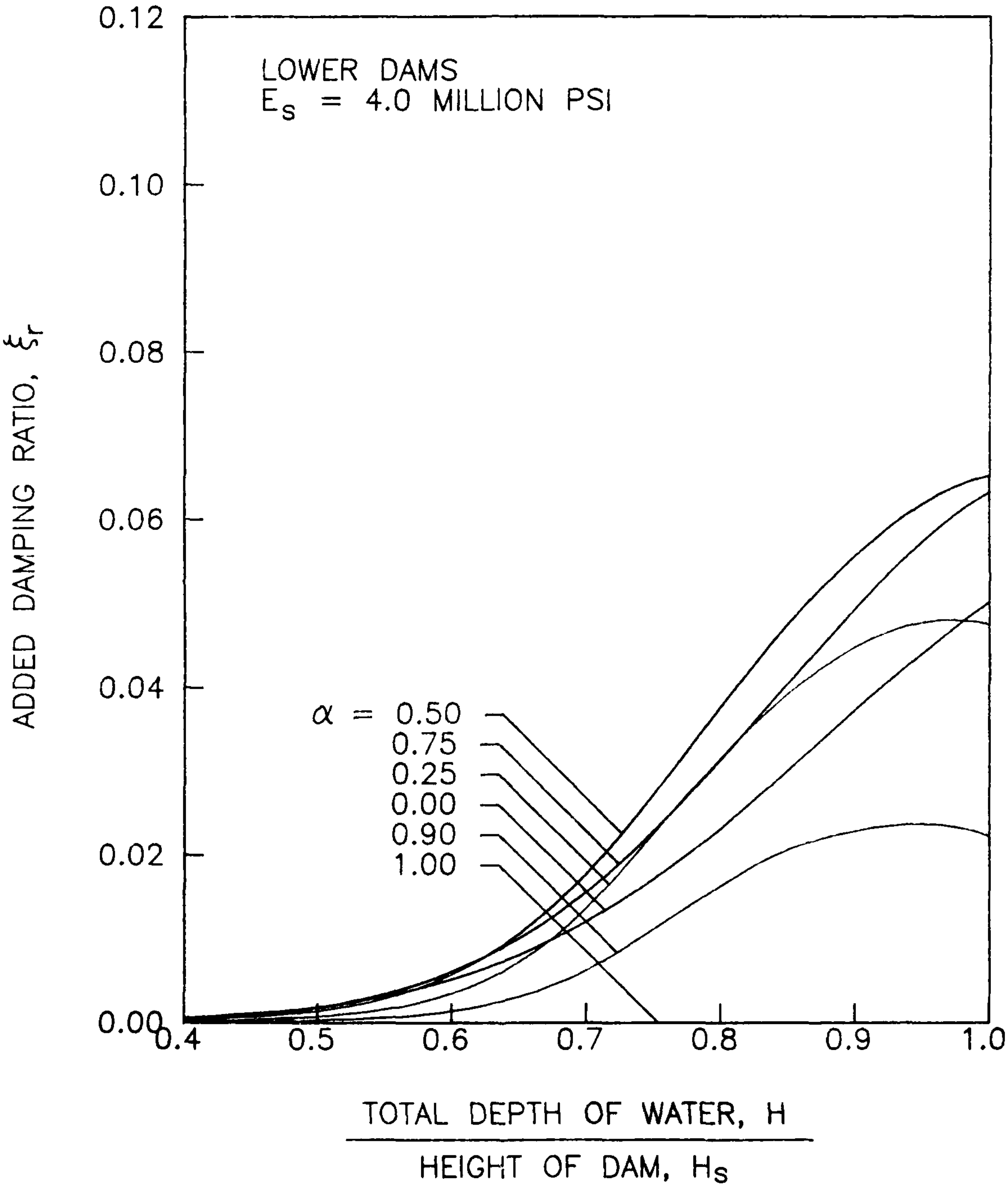

Figure 7(c) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 4 million psi.

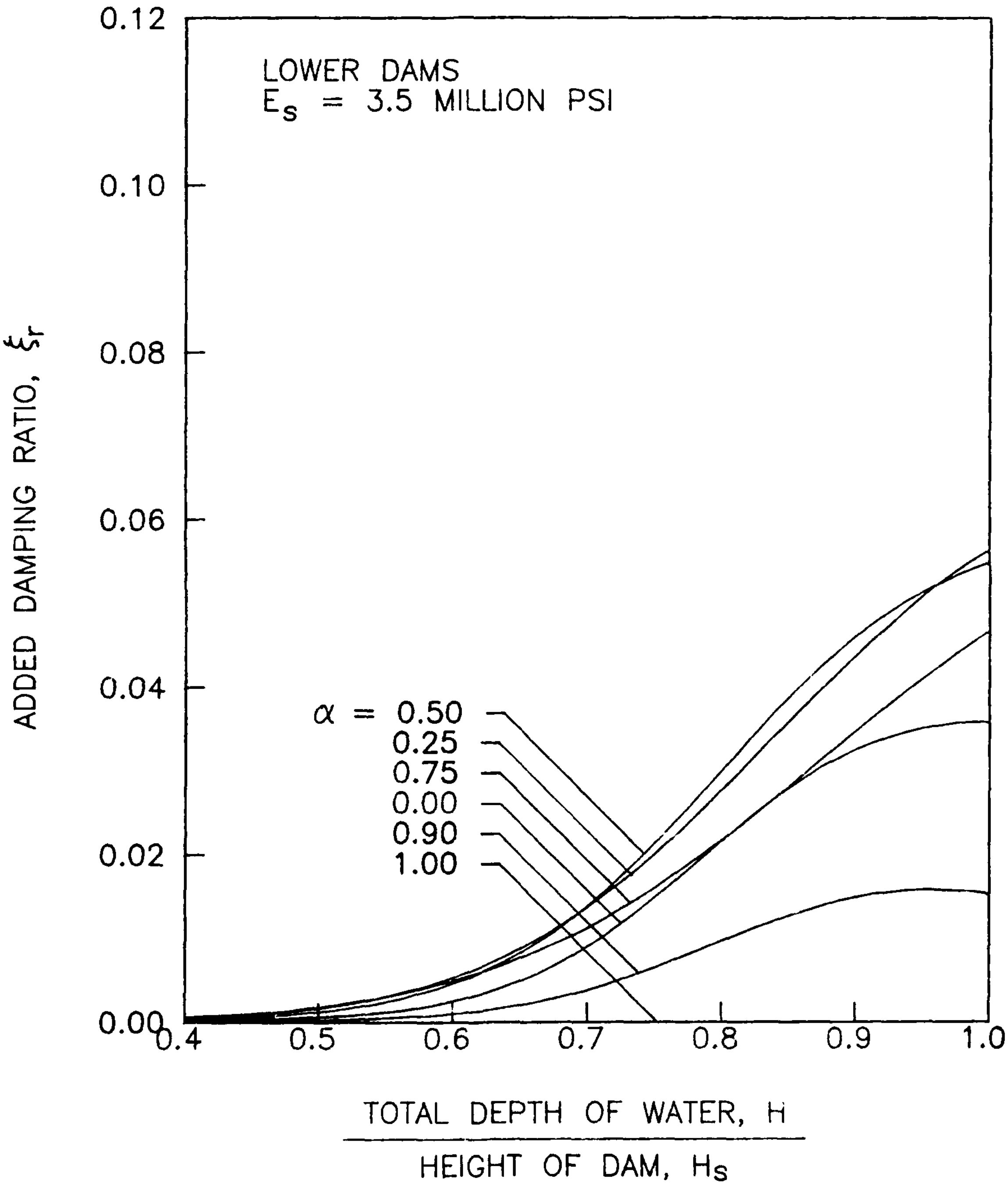

Figure 7(d) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 3.5 million psi.

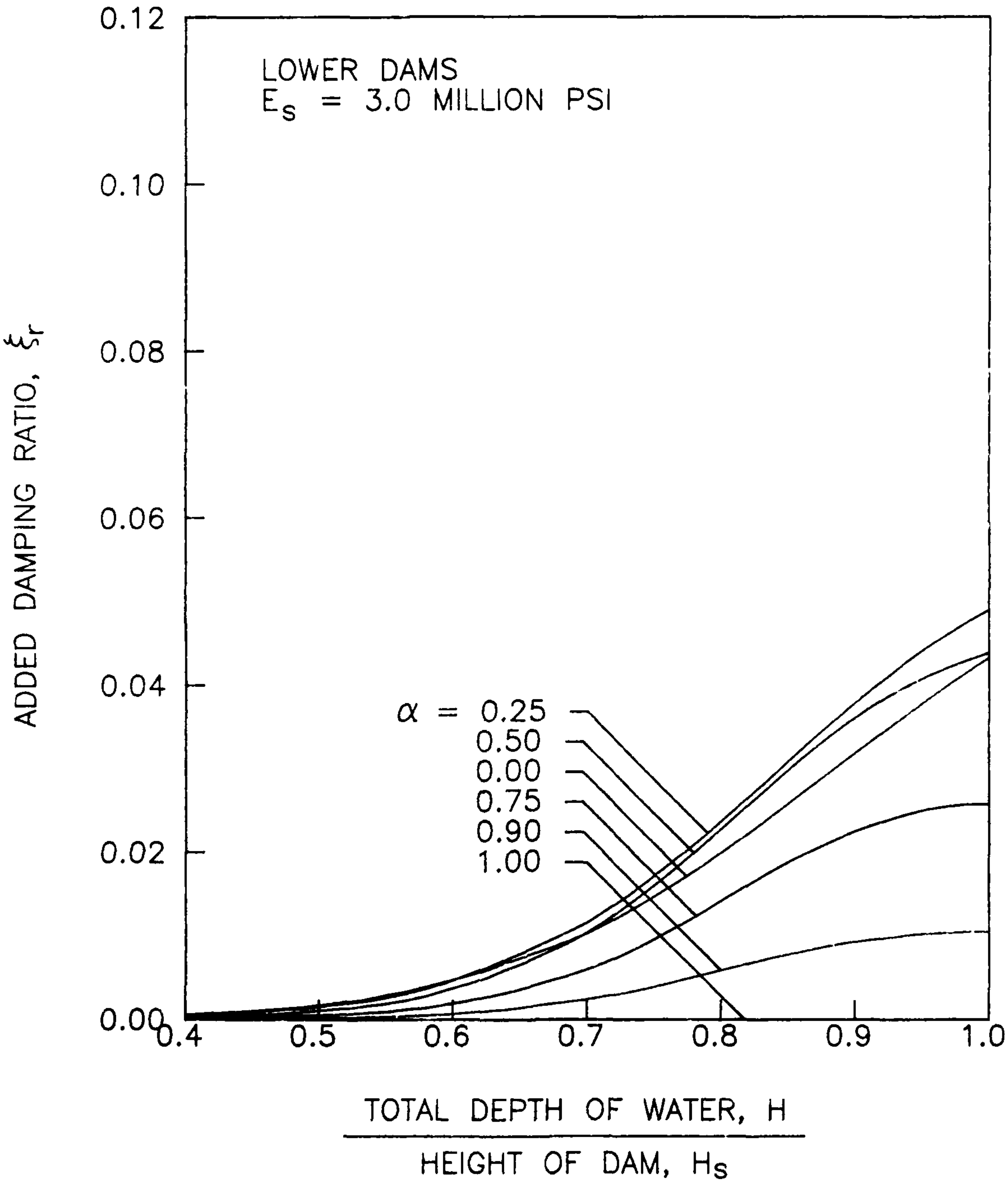

Figure 7(e) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 3 million psi.

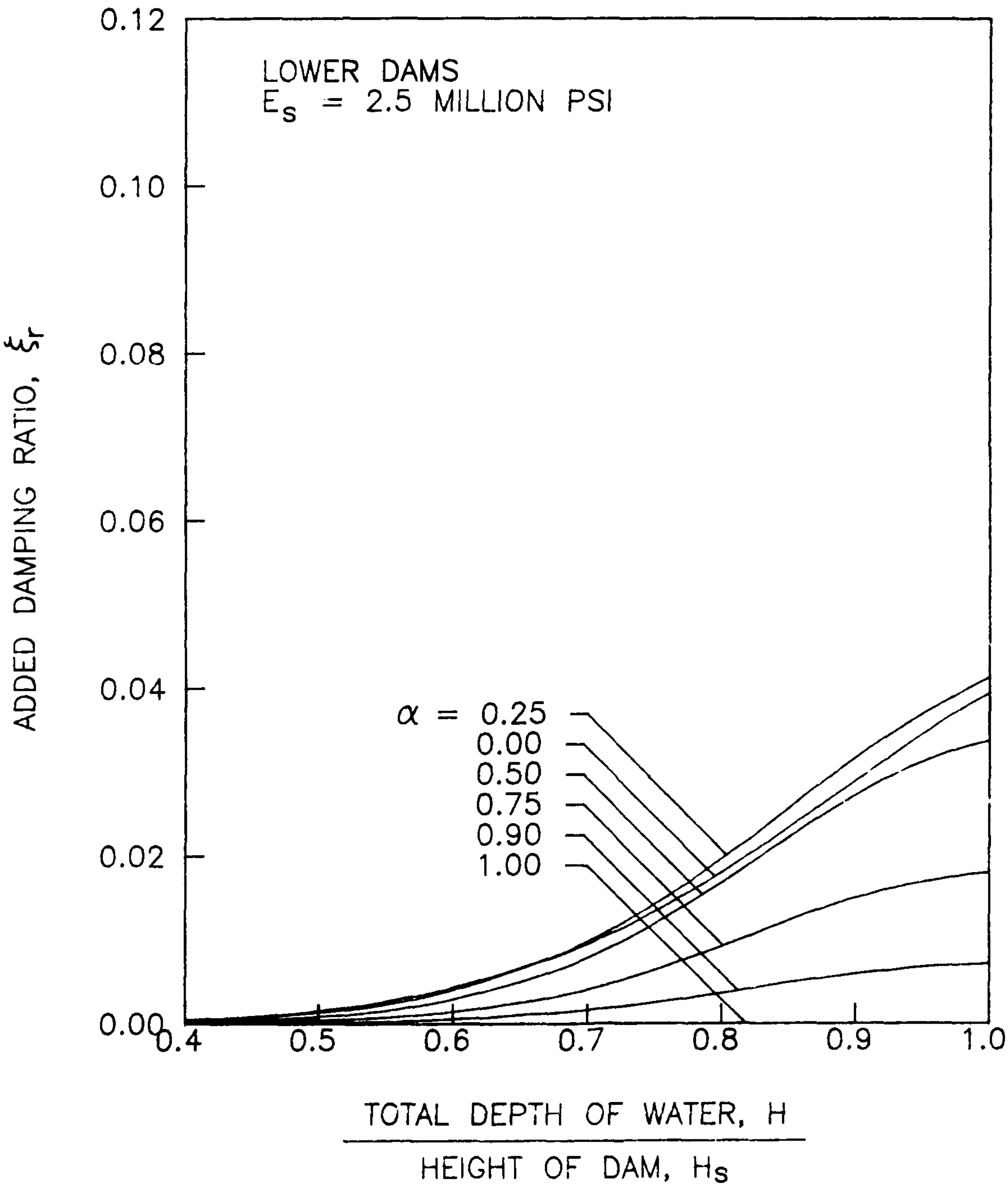

Figure 7(f) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 2.5 million psi.

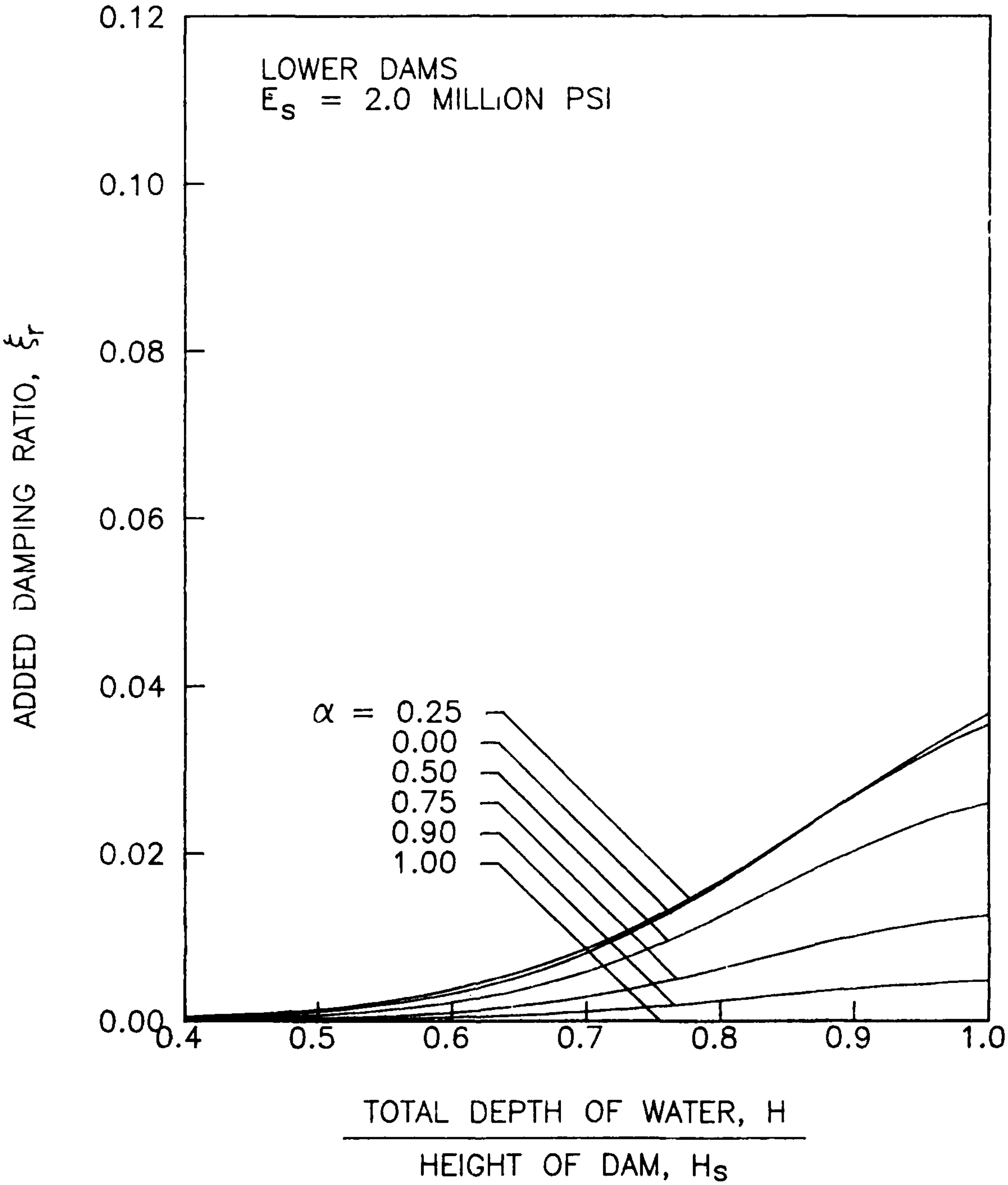

Figure 7(g) -- Standard Values for ξ_r, the Added Damping Ratio due to Hydrodynamic Effects; E_s = 2 million psi.

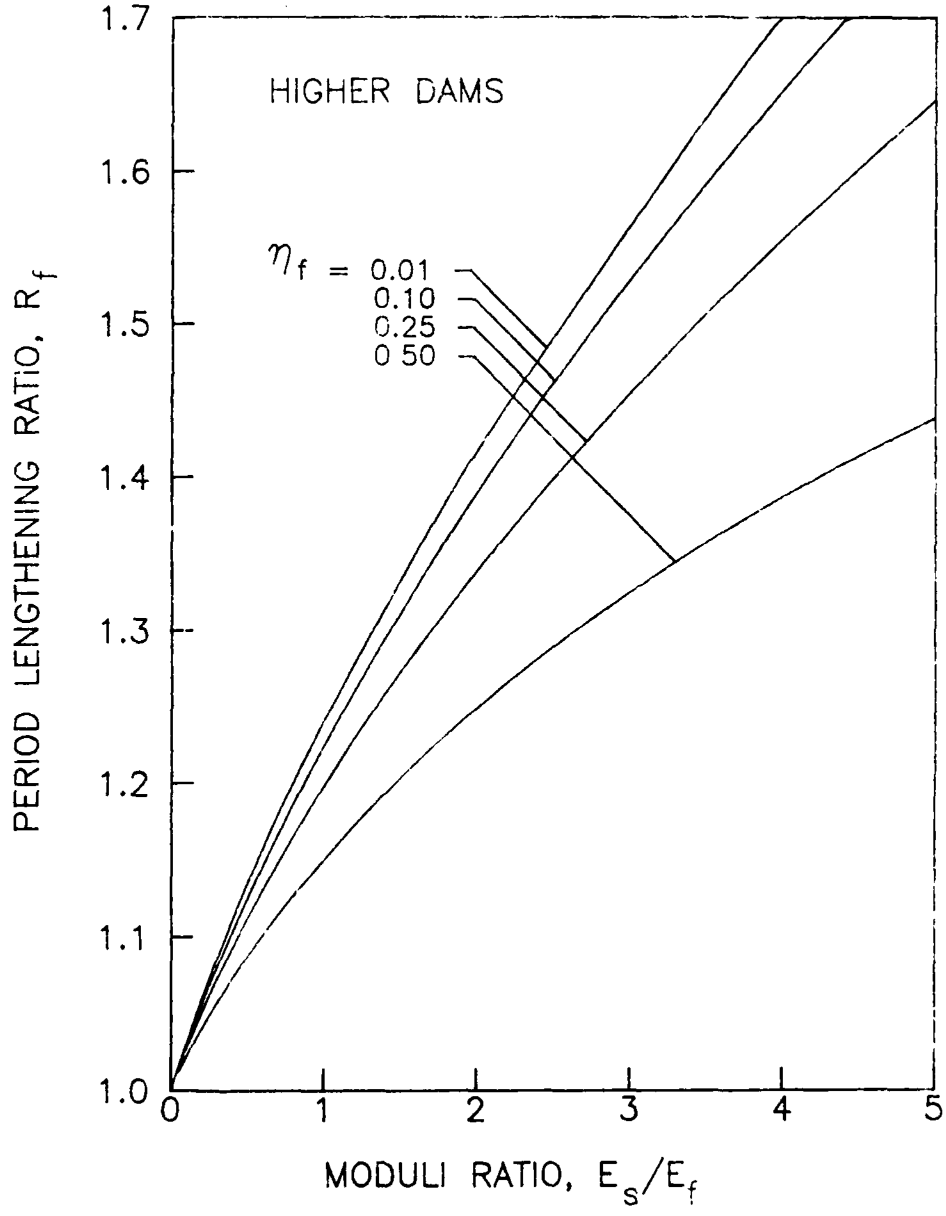

Figure 8 -- Standard Values for R_f, the Period Lengthening Ratio due to Dam–Foundation Rock Interaction

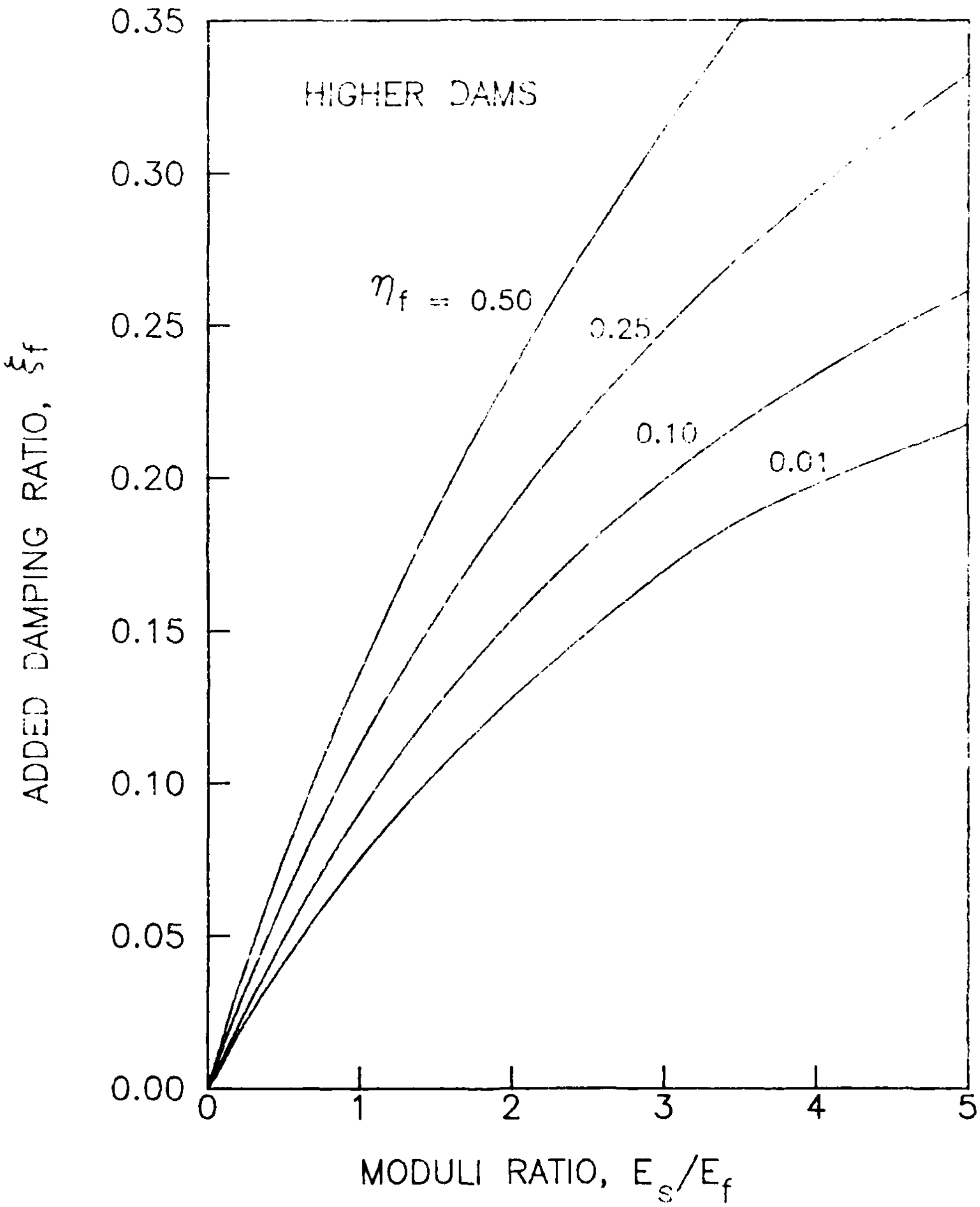

Figure 9 -- Standard Values for ξ_f, the Added Damping Ratio due to Dam–Foundation Rock Interaction

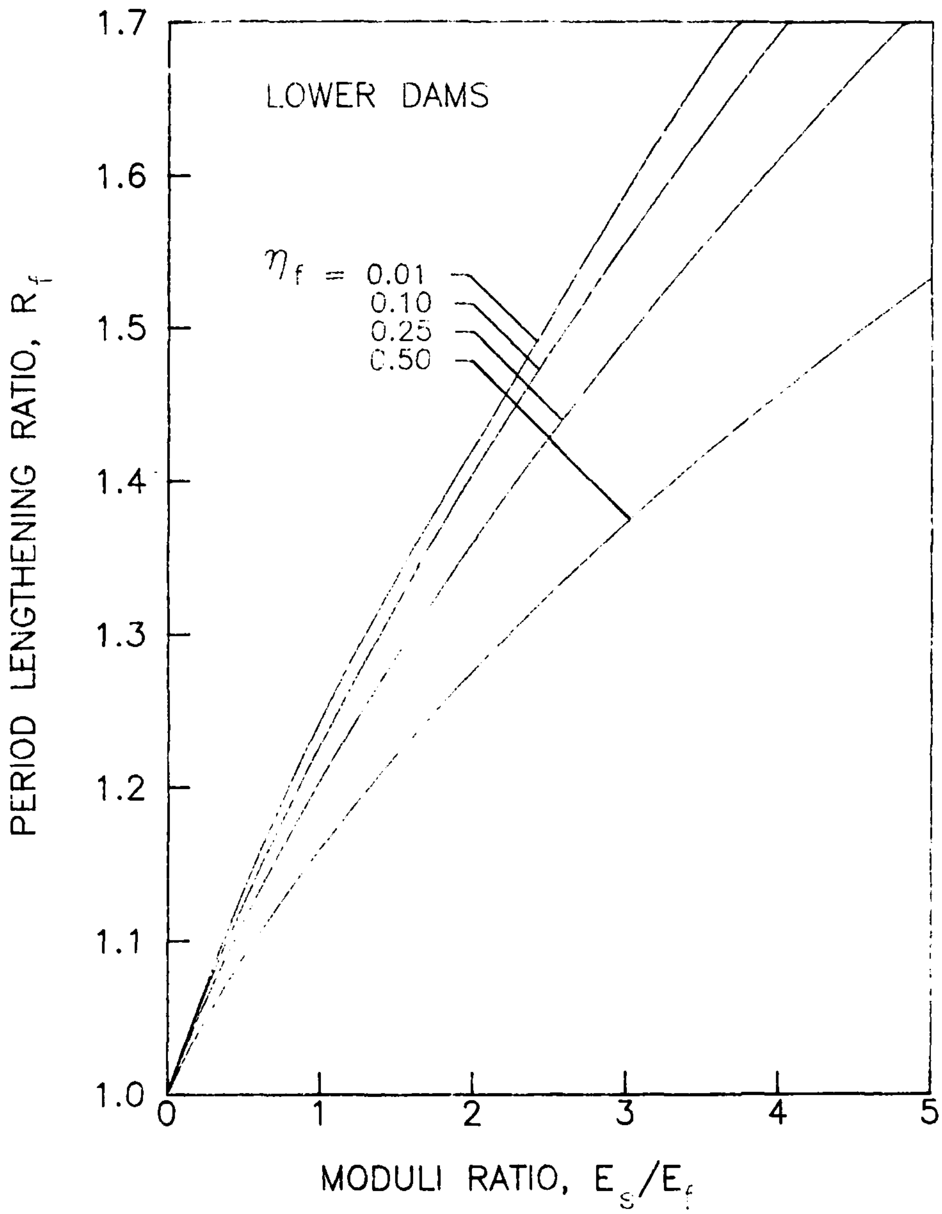

Figure 10 --- Standard Values for R_f, the Period Lengthening Ratio due to Dam–Foundation Rock Interaction

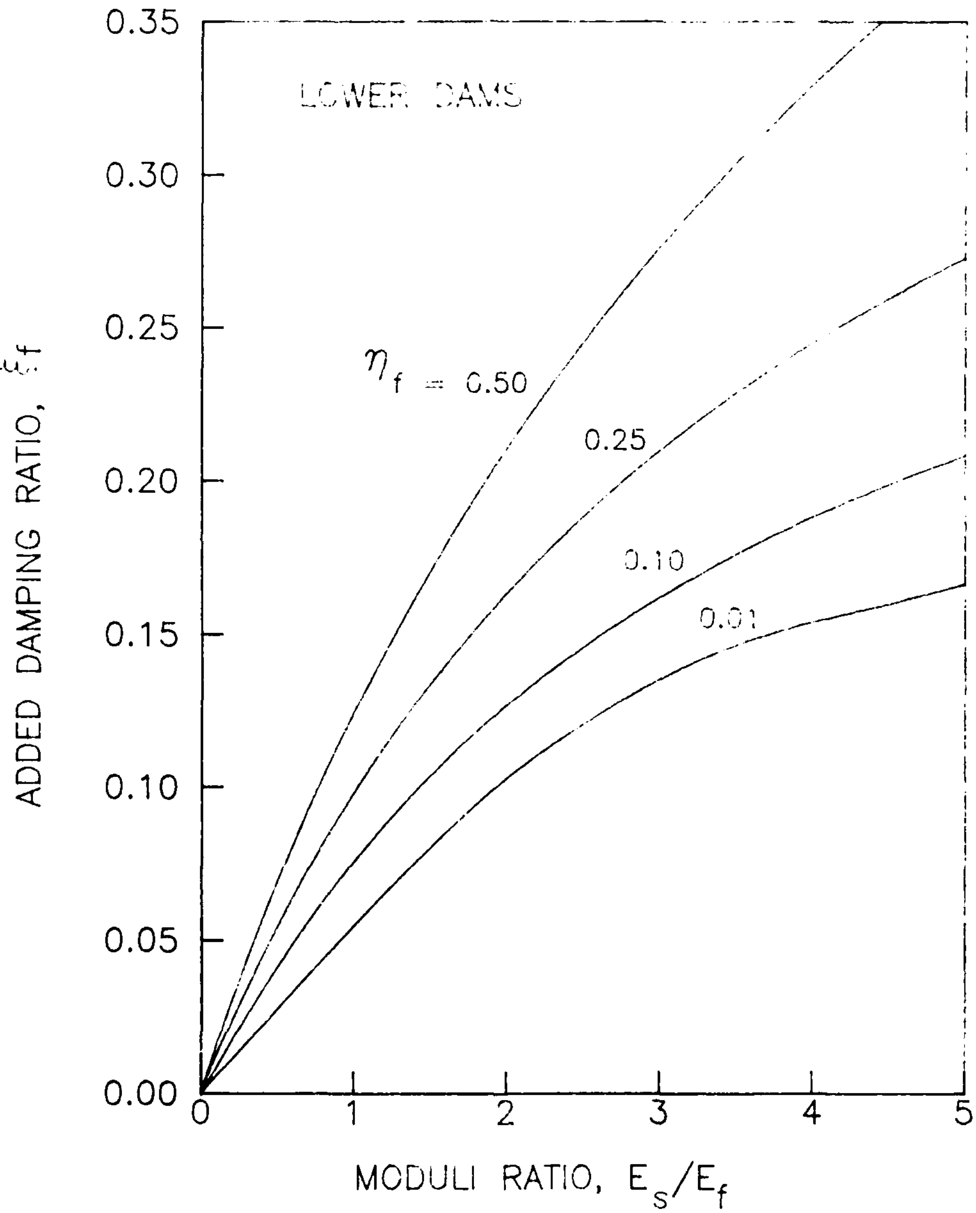

Figure 11 -- Standard Values for ξ_f, the Added Damping Ratio due to Dam-Foundation Rock Interaction

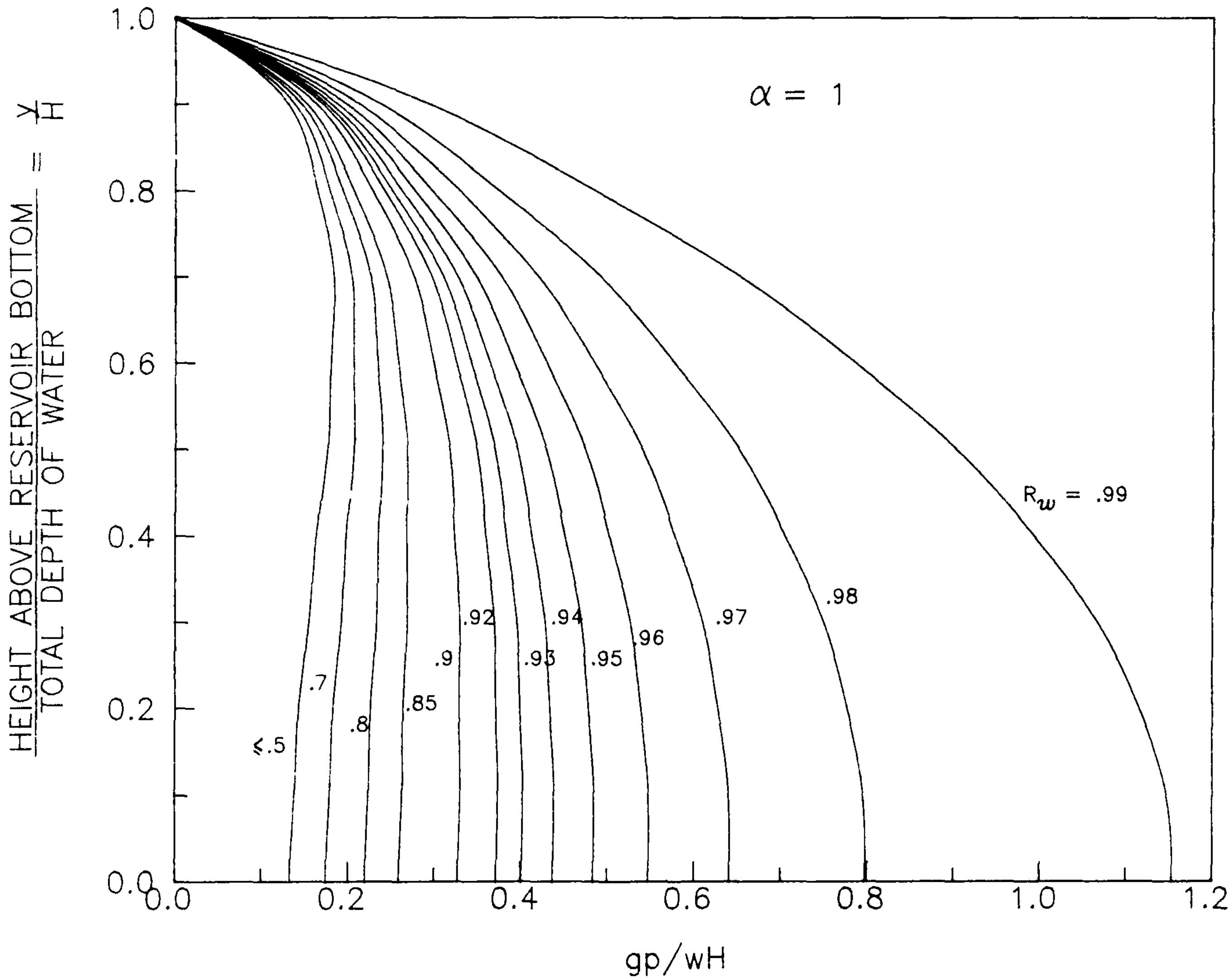

Figure 12(a) -- Standard Values for the Hydrodynamic Pressure Function $p(y/H)$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 1.00$ -- Higher and Lower Dams

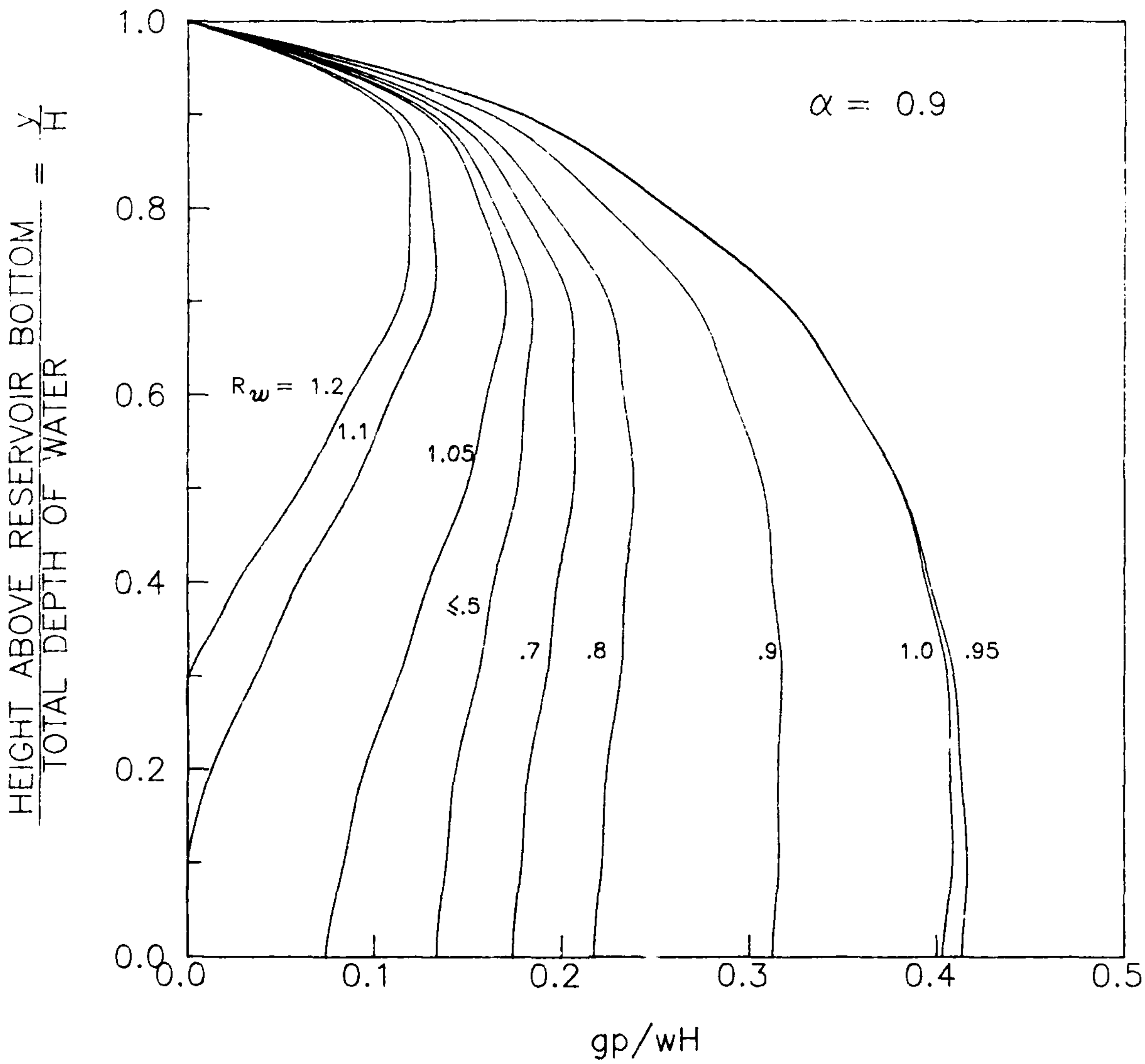

Figure 12(b) -- Standard Values for the Hydrodynamic Pressure Function $p(y/H)$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.90$ -- Higher and Lower Dams

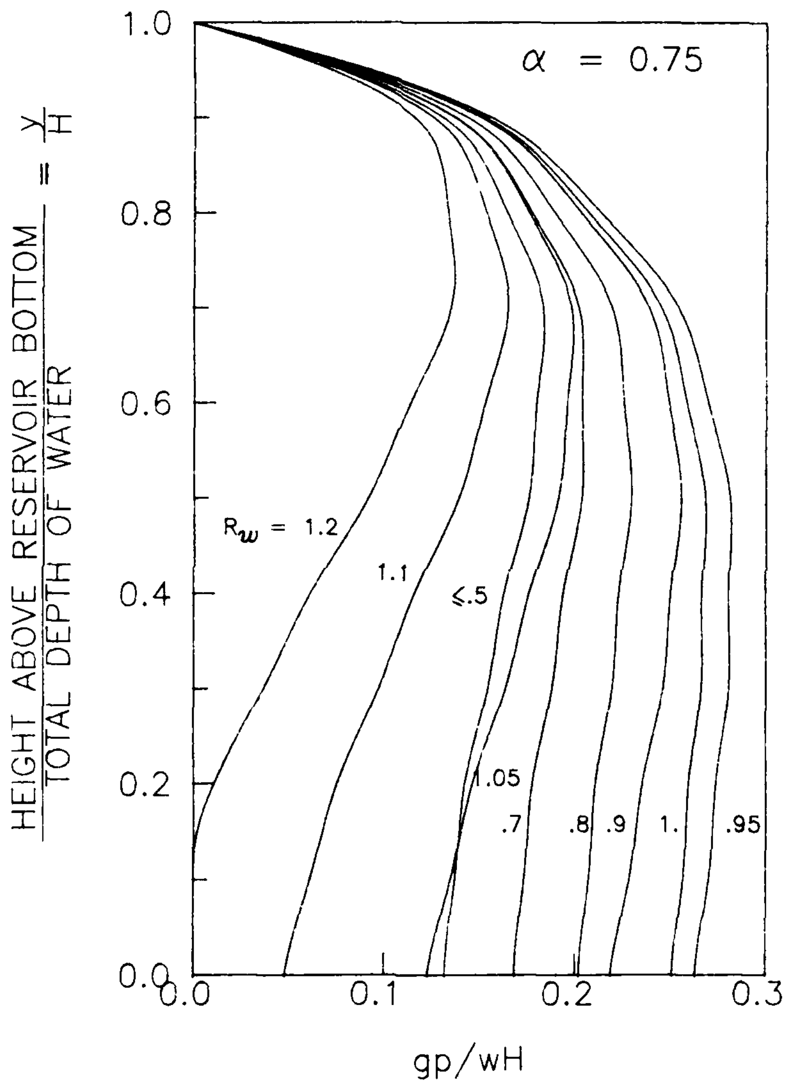

Figure 12(c) -- Standard Values for the Hydrodynamic Pressure Function $p(y/H)$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.75$ -- Higher and Lower Dams

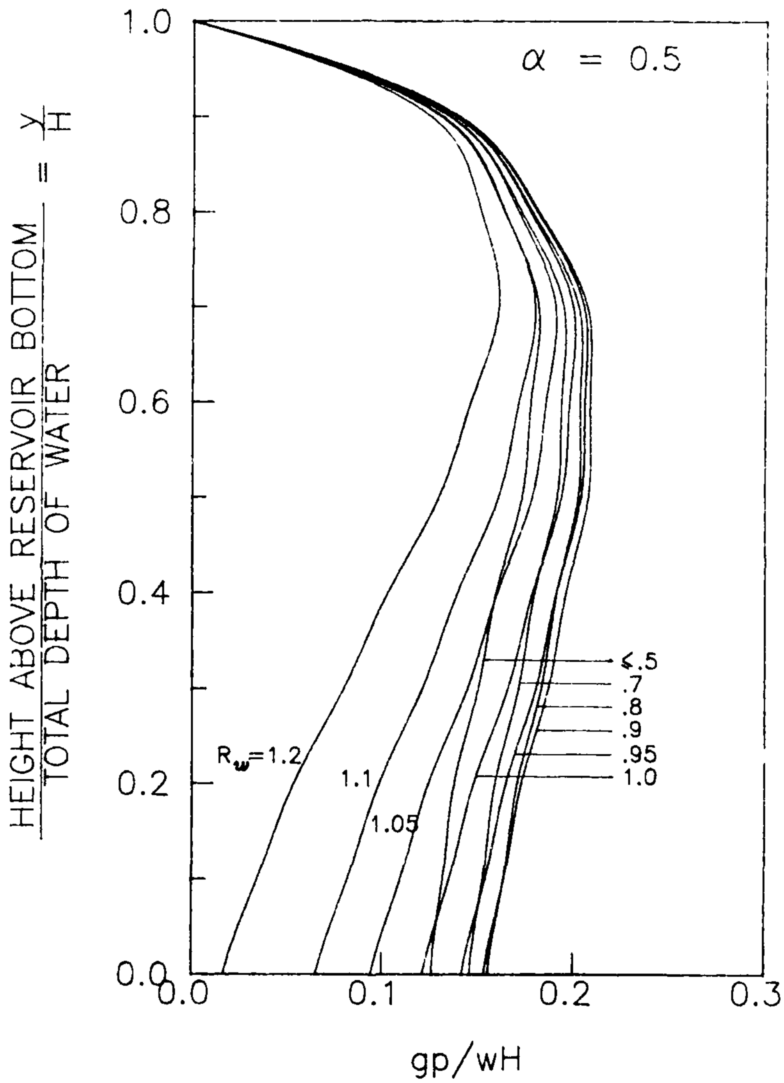

Figure 12(d) -- Standard Values for the Hydrodynamic Pressure Function $p(y/H)$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.50$ -- Higher and Lower Dams

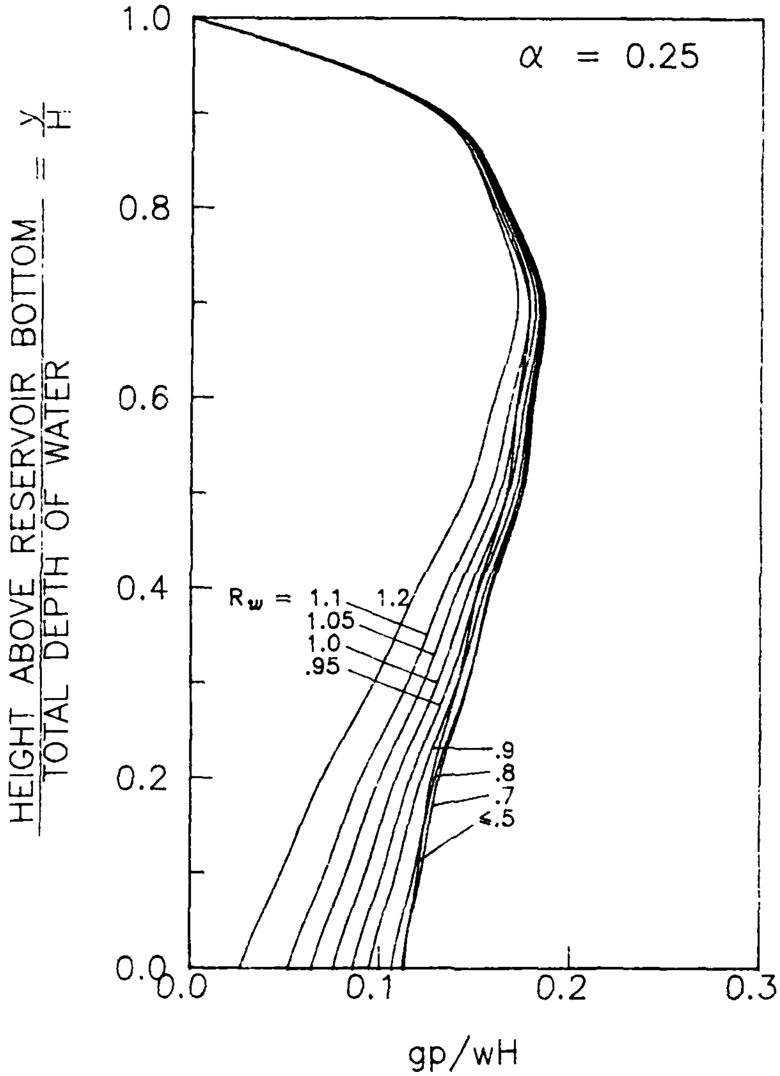

Figure 12(e) -- Standard Values for the Hydrodynamic Pressure Function $p(y/H)$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.25$ -- Higher and Lower Dams

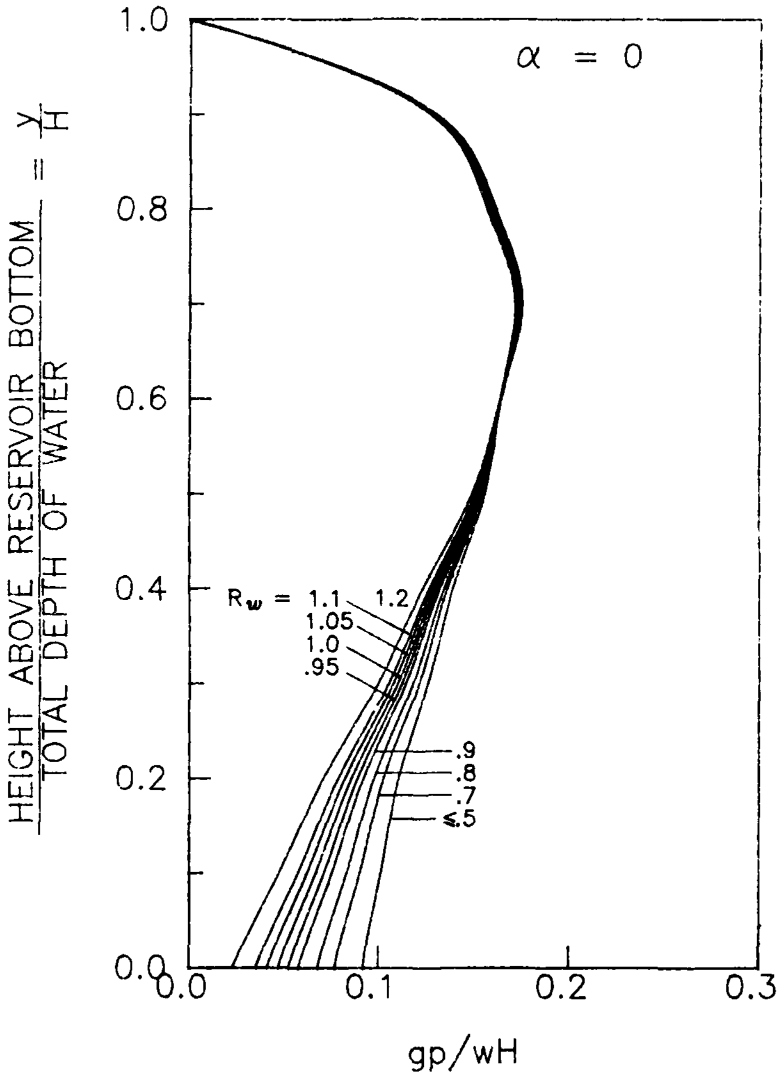

Figure 12(f) -- Standard Values for the Hydrodynamic Pressure Function $p(y/H)$ for Full Reservoir, i.e., $H/H_s = 1$; $\alpha = 0.00$ -- Higher and Lower Dams

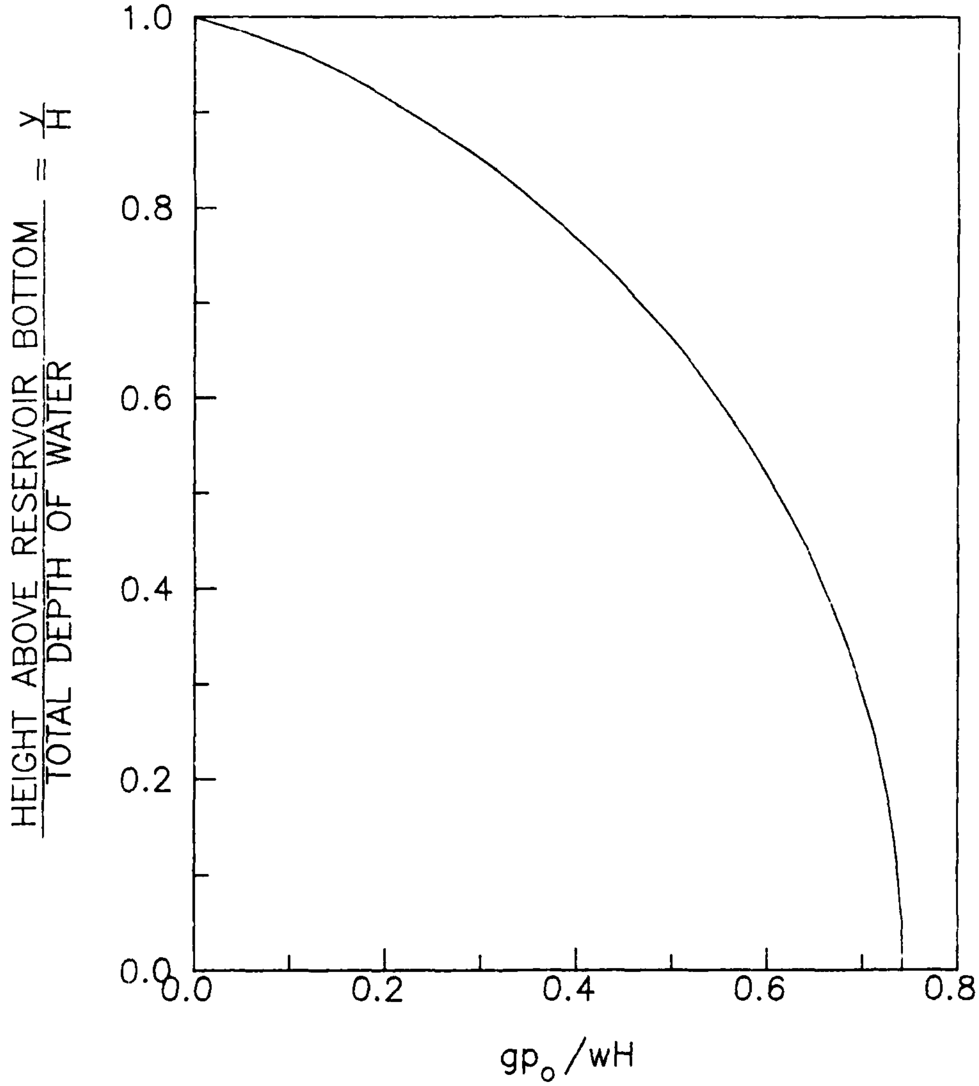

Figure 13 -- Standard Values for the Hydrodynamic Pressure Function $p_o(y)$

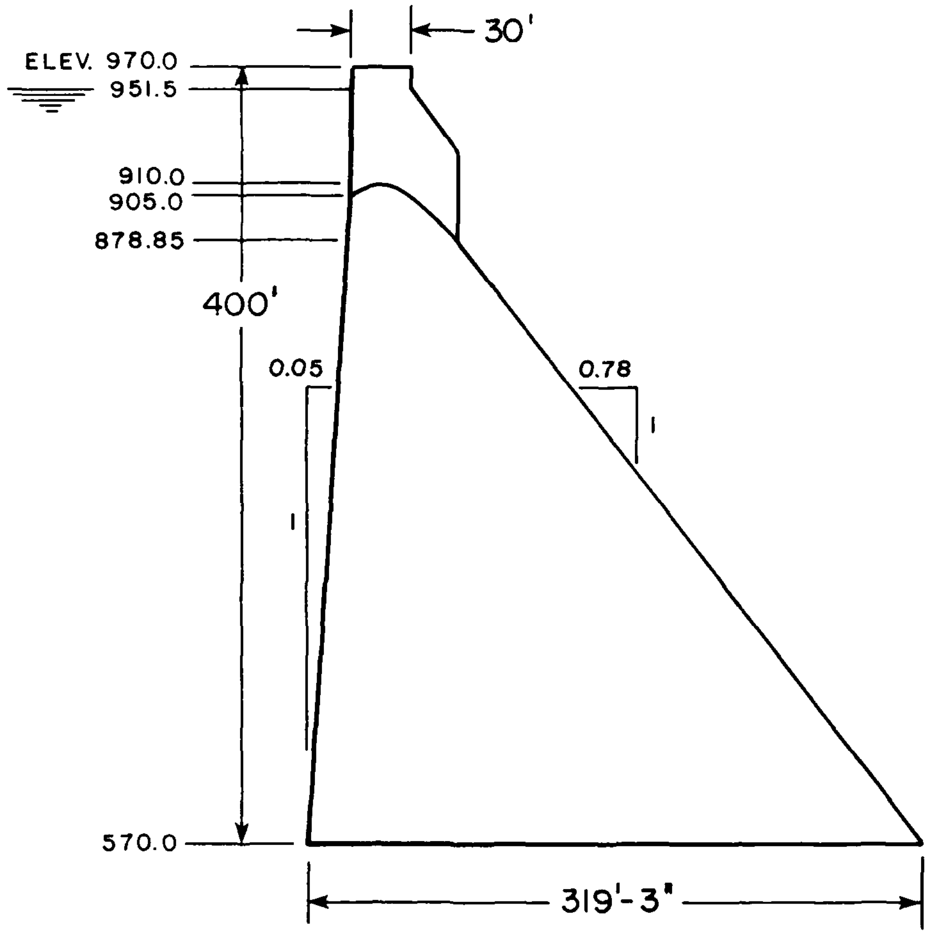

Figure 14 -- Pine Flat Dam: Tallest Spillway Monolith and Pier

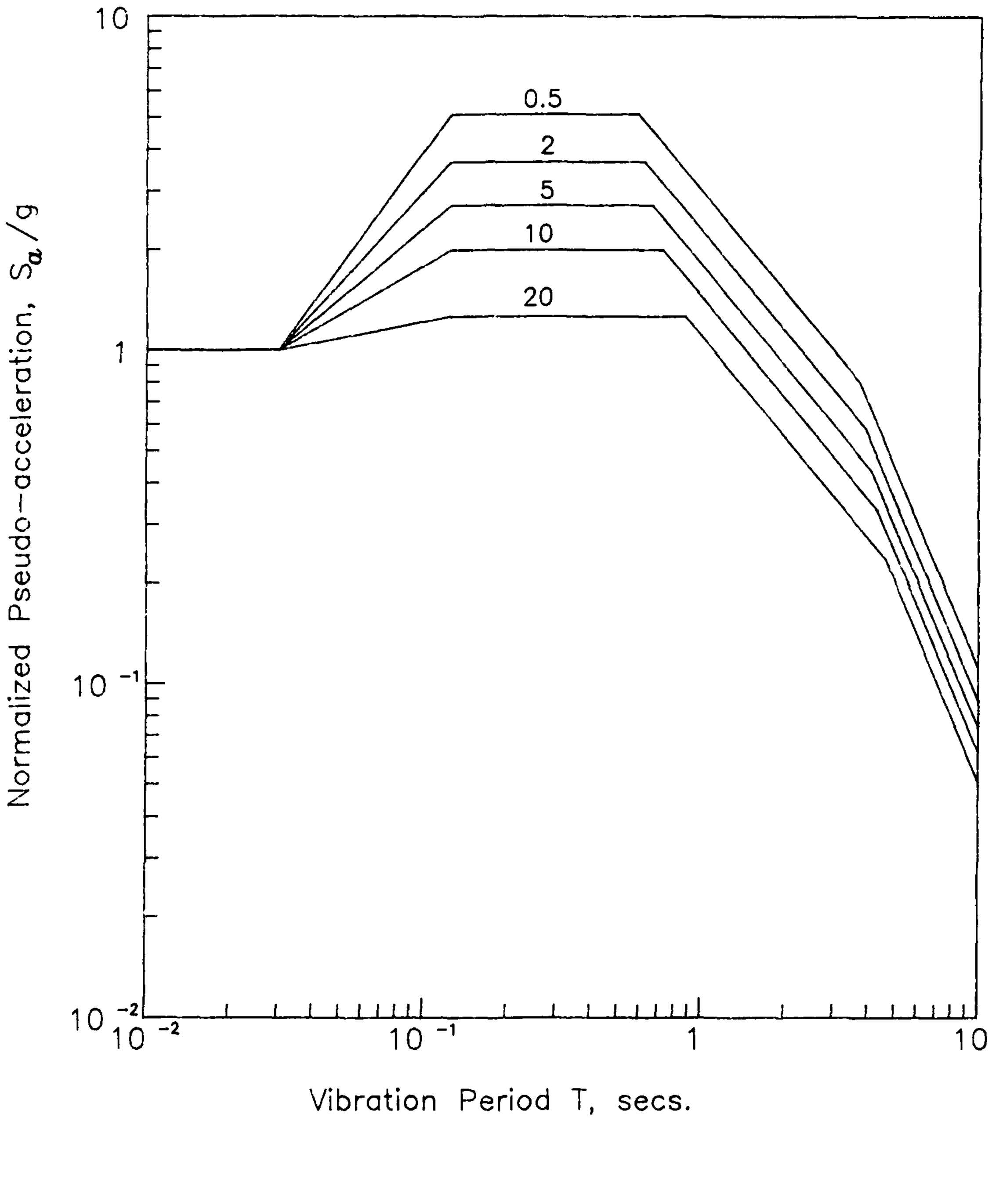

Figure 15 -- Elastic Design Spectrum, Horizontal Motion, One Sigma Cumulative Probability, Damping Ratios = 0.5, 2, 5, 10, and 20 percent

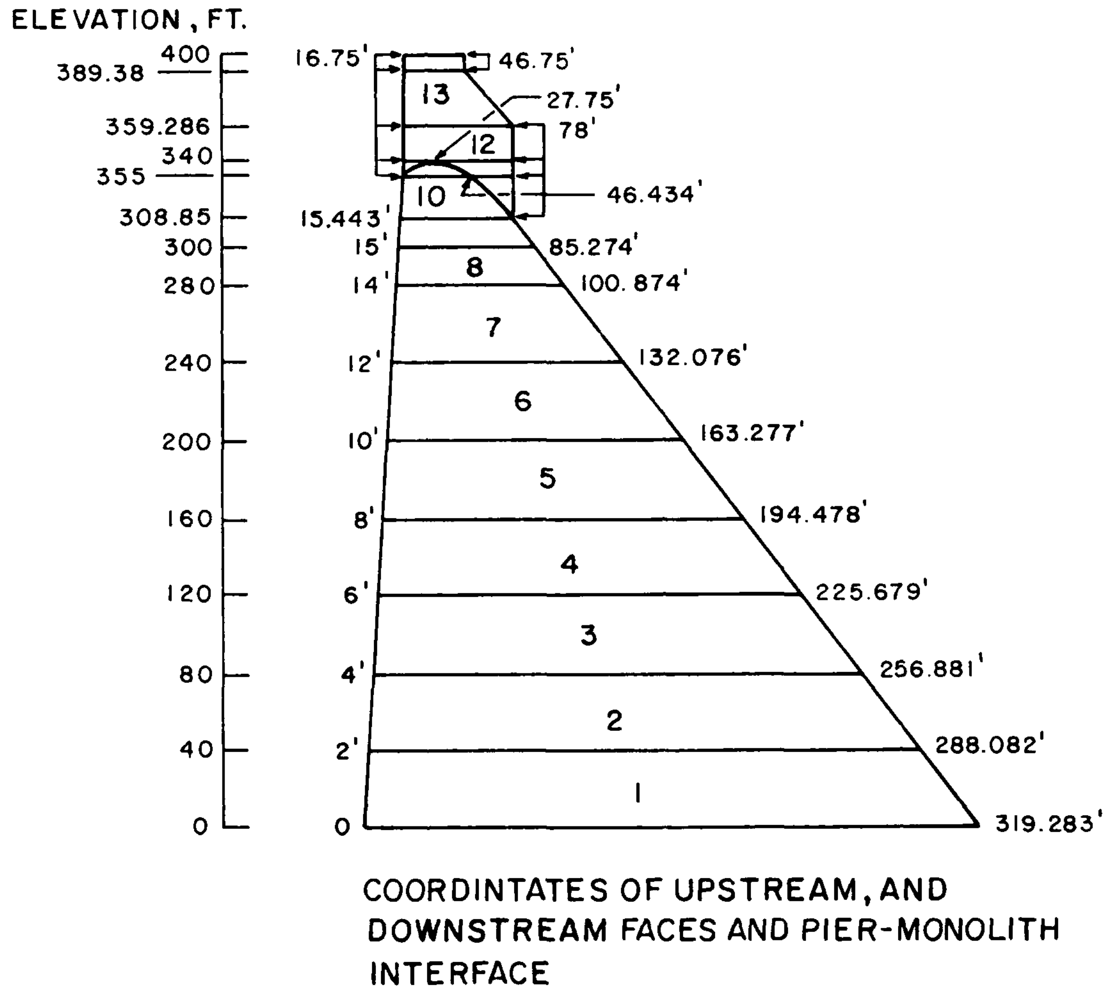

Figure 16 -- Block Model of Spillway of Pine Flat Dam

APPENDIX A: STANDARD SPILLWAY CROSS-SECTIONS

The spillway monoliths of Pine Flat Dam and Richard B. Russel Dam are idealized as shown in Fig. A.1. Computations carried out at the writer's request by Dr. R. L. Hall, Waterways Experiment Station, Corps of Engineers, have demonstrated that the bridge, gate, and foot bucket may be neglected in estimating the fundamental vibration period and mode shape. Therefore, these components have not been included in the structural idealization. Also, the pier geometry has been somewhat simplified in the idealization. The height H_s is 400 ft for the tallest monolith of Pine Flat Dam (Fig. A.1(a)) and 200 ft in the case of Russell Dam (Fig. A.1(b)). However, the cross-section of Fig. A.1(a) is representative of higher dams, i.e. H_s = 300 to 600 ft, and Fig. A.1(b) is typical of lower dams, i.e. H_s = 0 to 300 ft.

The fundamental vibration period and mode shape of dams (monolith plus pier) of Fig. A.1 were computed for various values of H_s, with the size and shape of the pier unchanged. Because the widths of the monolith and pier along the dam axis are 50 ft and 10 ft, respectively, it may seem that a three-dimensional analysis is required. However, computations carried out by Dr. R. L. Hall at the writer's request led to the following conclusion: An equivalent two-dimensional system of unit thickness along the dam axis, with the unit weight and elastic modulus of the pier reduced by a factor equal to the ratio of the monolith width to pier width, is satisfactory for computing the fundamental vibration period and mode shape of a dam. Finite element idealizations of these two-dimensional systems were analyzed by the EAGD-84 computer program. In these analyses the Poisson's ratio of the dam concrete was taken as 0.2.

The resulting fundamental vibration periods and mode shapes are presented in Fig. A.2 for dams of Fig. A.1(a) for four heights H_s between 300 and 600 ft. Expressing the computed fundamental vibration period as

$$T_1 = \beta \frac{H_s}{\sqrt{E}} \tag{A.1}$$

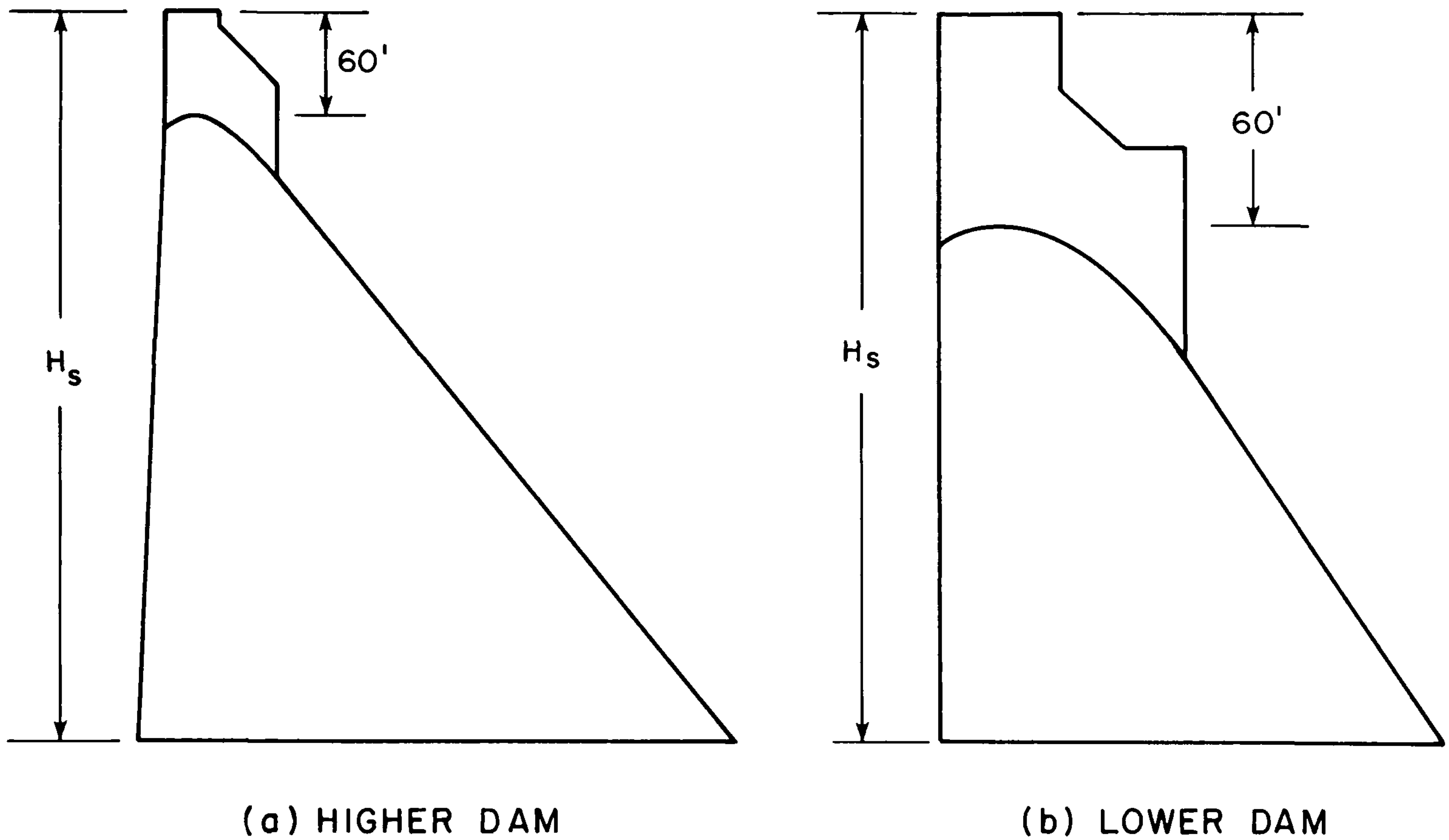

Figure A1 -- Idealized Spillway Monoliths

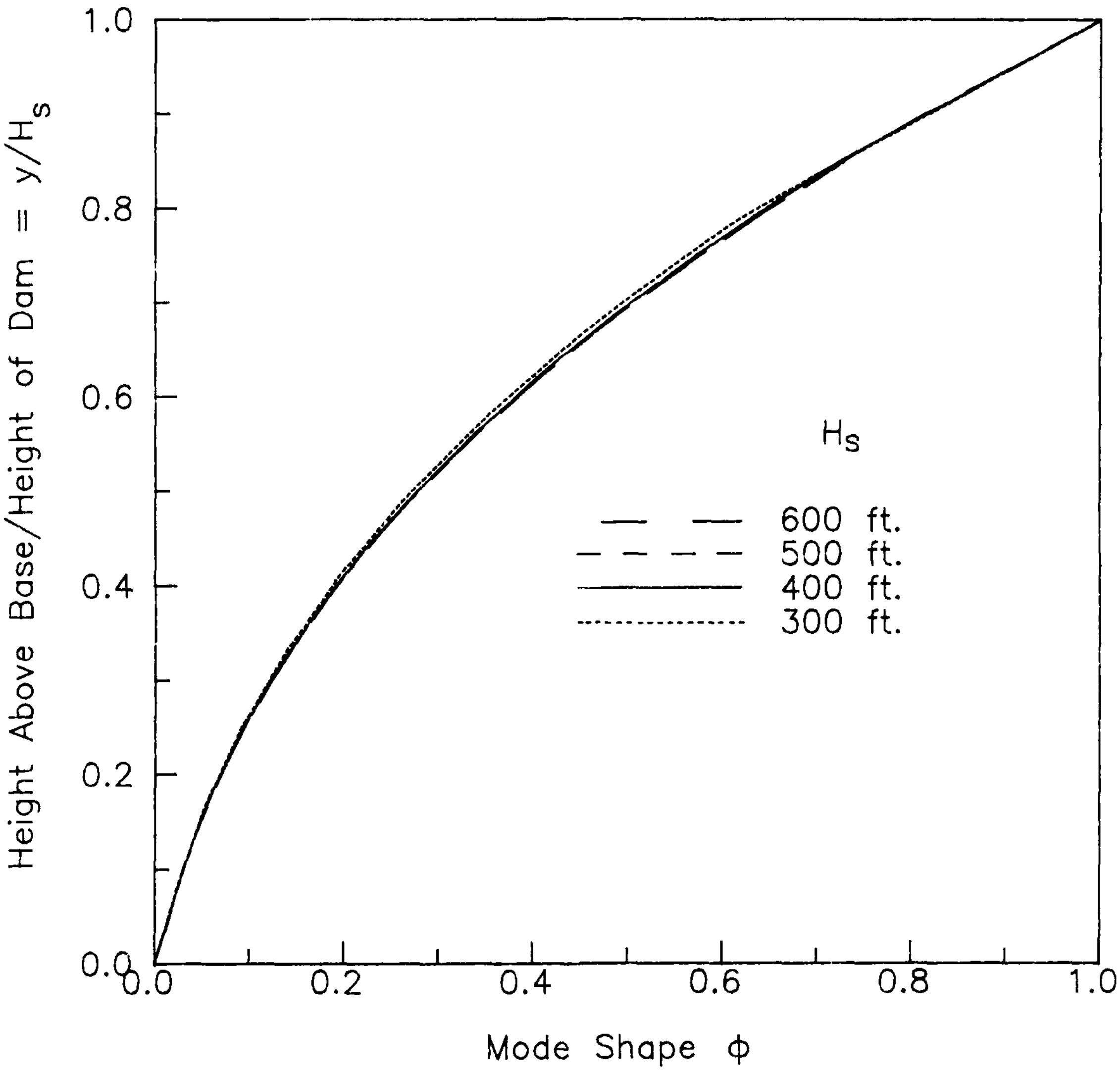

Figure A2 -- Fundamental Mode Shape of Spillway Monolith of Fig. A1(a) for Various Values of H_s

the values of β were determined to be 1.22, 1.21, 1.20, and 1.17 for dams with H_s = 600, 500, 400, and 300 ft, respectively. For the dams of Fig. A.1(b) the vibration mode shape is presented in Fig. A.3 and the associated values of β are 1.33, 1.29, 1.23, and 1.17 for H_s = 300, 250, 200, and 150 ft. Note that all the mode shapes of Fig. A.2 (or Fig. A.3) and the associated β values would have been identical if the pier height was not fixed and was proportional to H_s.

Based on these results, the standard properties for higher dams ($H_s \geq 300$ ft) are based on the dam of Fig. A.1(a) with H_s = 400 ft. Many analyses of this "standard" spillway cross-section were carried out to obtain the standard data presented in this report for higher dams to be used in conjunction with the simplified anlaysis procedure. In particular, the standard value for β is selected to be 1.2 (Eq. 6a) and the standard mode shape is presented in Fig. 3(a) and Table 1(a).

Similarly, the standard properties for lower dams ($H_s < 300$ ft) are based on the dam of Fig. A.1(b) with H_s = 200 ft. Many analyses of this "standard" spillway cross-section were carried out to obtain the standard data presented in this report for lower dams. In particular, the standard value for β is selected to be 1.25 (Eq. 6b) and the standard mode shape is presented in Fig. 3(b) and Table 1(b).

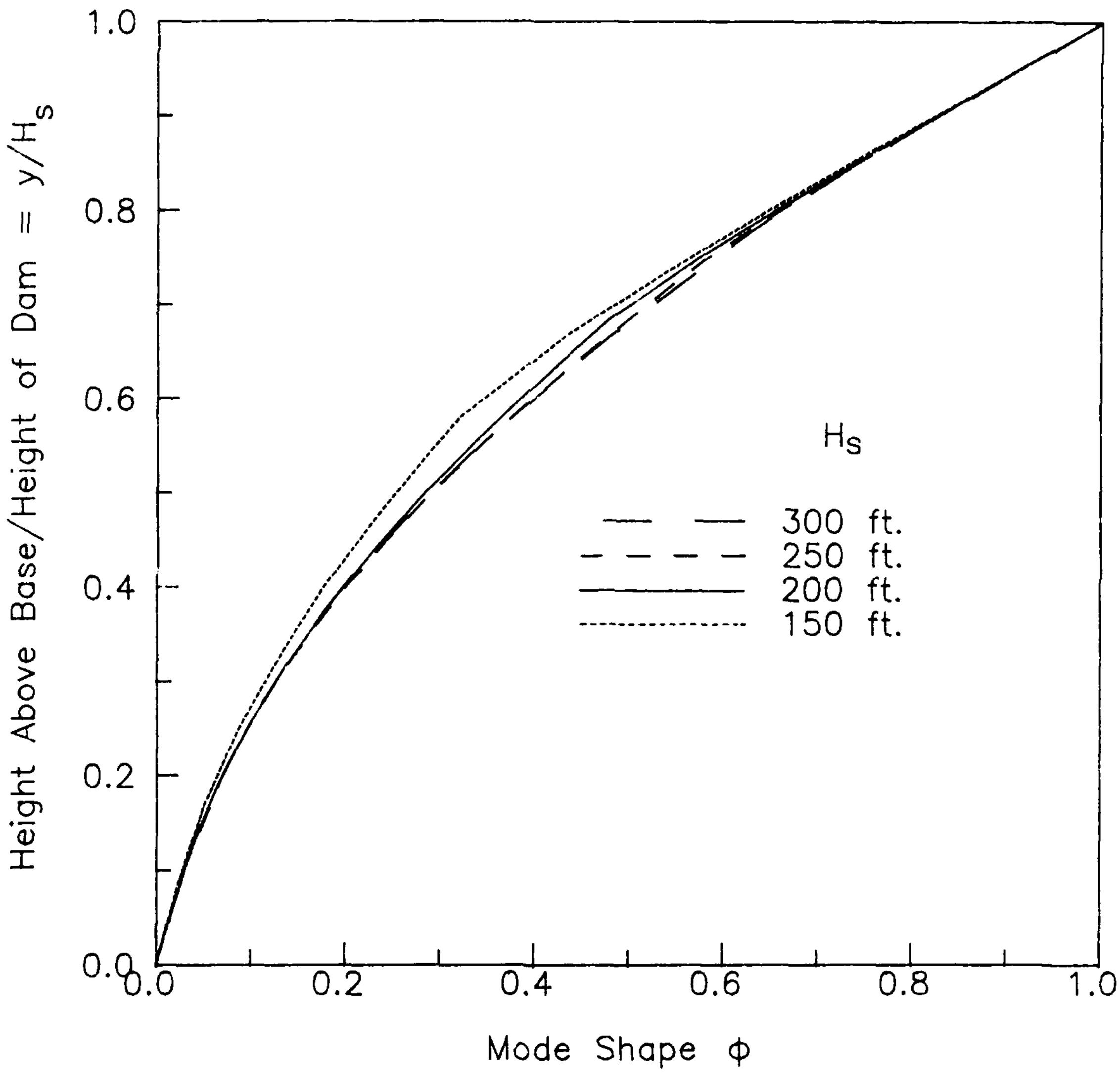

Figure A3 -- Fundamental Mode Shape of Spillway Monolith of Fig. A1(b) for Various Values of H_s

APPENDIX B: DETAILED CALCULATIONS FOR PINE FLAT DAM

This appendix presents the detailed calculations required in the simplified analysis procedure as applied to the tallest, gated spillway monolith of Pine Flat Dam. All computations are performed for the equivalent two-dimensional system of unit thickness representing the dam (see page 3). Only the details for Case 4 in Table 9 (full reservoir and flexible foundation rock) are presented.

Simplified Model of Monolith and Pier

The tallest gated spillway monolith of Pine Flat Dam is divided into fourteen blocks as shown in Fig. 16. Using a unit weight of 155 pcf for the concrete and the ratio of pier width to monolith width = 0.16 in defining $w_s(y)$ for the pier (page 3), the properties of the blocks are presented in Table B.1, from which the total weight is 9434 kips. Replacing the integrals in Eqs. 2b and 3b by the summations over the blocks gives:

$$M_1 \approx \frac{1}{g} \sum_{i=1}^{10} w_i \phi^2(y_i) = \frac{1}{g}(559 \text{ kip}) \tag{B.1}$$

$$L_1 \approx \frac{1}{g} \sum_{i=1}^{10} w_i \phi(y_i) = \frac{1}{g}(1623 \text{ kip}) \tag{B.2}$$

where w_i and y_i are the weight of block i and the elevation of its centroid, respectively. Additional properties of the simplified model are listed in Table B.2.

Equivalent Lateral Forces -- Fundamental Mode

The equivalent lateral earthquake forces $f_1(y)$ are given by Eq. 1, evaluated at each level using $S_a(\tilde{T}_1, \tilde{\xi}_1)/g$ = 0.377 (from Table 9 and Fig. 15) and $\tilde{L}_1/\tilde{M}_1$ = 3.14 (from Step 8 in the simplified procedure). The calculations are summarized in Table B.3.

Table B.1 -- Properties of the Simplified Model

Block	Weight w (k)	Elevation of centroid (ft)	ϕ (1) at centroid	$w\phi$ (k)	$w\phi^2$ (k)
14	7.9	394.7	0.976	7.7	7.5
13	34.1	372.6	0.875	29.8	26.1
12	29.3	349.6	0.769	22.5	17.3
11	17.3	337.0	0.713	12.3	8.8
10	197.2	320.7	0.649	127.9	83.0
9	91.1	304.3	0.588	53.6	31.5
8	243.6	289.6	0.537	130.7	70.2
7	641.5	258.9	0.437	280.2	122.4
6	847.4	219.2	0.325	275.4	89.5
5	1053.	179.3	0.231	242.9	56.0
4	1259.	139.5	0.154	194.2	29.9
3	1465.	99.5	0.093	136.8	12.8
2	1671.	59.6	0.048	79.6	3.8
1	1877.	19.6	0.016	29.4	0.5
Total	9434			1623	559

(1) From Fig. 3(a) or Table 1(a).

Table B.2 -- Additional Properties of the Simplified Model

Level	Elevation y (ft)	Width of Monolith $b^{(1)}$ (ft)	Width of Pier b_p (ft)	Weight per Unit Height $w_s^{(2)}$ (k/ft)	Section Modulus $S=1/6\ b^2$ (3) (ft^3)
Top	400.	-	30.0	0.74	-
14	389.38	-	30.0	0.74	-
13	359.286	-	61.3	1.52	-
12	340.	0	61.3	1.52	-
11	335.	29.7	31.6	5.39	-
10	308.85	62.6	0	9.70	653.1
9	300.	70.3	-	10.90	823.7
8	280.	86.9	-	13.47	1259.
7	240.	120.1	-	18.62	2404.
6	200.	153.3	-	23.72	3917.
5	160.	186.5	-	28.91	5797.
4	120.	219.7	-	34.05	8045.
3	80.	252.9	-	39.20	10660.
2	40.	286.1	-	44.35	13640.
1	0.	319.3	-	49.49	16990.

(1) From Fig. 16.

(2) $w_s = 0.155\ b$ for monolith blocks

$= 0.155 \times 0.16\ b_p$ for pier blocks

$= 0.155 \times (\ b + 0.16\ b_p\ )$ for transition blocks

(3) Computed only for monolith blocks, as the pier should be analyzed as a reinforced concrete structure.

Table B.3 -- Equivalent Lateral Earthquake Forces -- Fundamental Vibration Mode

Level	y (ft)	w_s (1) (k/ft)	y/H_s	ϕ (2)	$w_s\phi$ (k/ft)	y/H	gp/wH (3)	gp (k/ft)	$f_1(y)$ (5) (k/ft)
Top	400.	0.74	1.0	1.0	0.74	1.05	0	0	0.87
14	389.38	0.74	0.97	0.95	0.70	1.02	0	0 (4)	0.83
13	359.286	1.52	0.90	0.81	1.23	0.94	0.092	1.98	3.81
12	340.	1.52	0.85	0.73	1.10	0.89	0.144	3.11	4.98
11	335.	5.39	0.84	0.71	3.81	0.88	0.151	3.26	8.36
10	308.85	9.70	0.77	0.61	5.87	0.81	0.178	3.84	11.49
9	300.	10.90	0.75	0.57	6.24	0.79	0.185	3.99	12.11
8	280.	13.47	0.70	0.50	6.79	0.74	0.199	4.29	13.12
7	240.	18.62	0.60	0.38	7.09	0.63	0.209	4.51	13.74
6	200.	23.72	0.50	0.28	6.57	0.53	0.209	4.51	13.12
5	160.	28.91	0.40	0.19	5.55	0.42	0.198	4.27	11.63
4	120.	34.05	0.30	0.12	4.19	0.32	0.189	4.08	9.79
3	80.	39.20	0.20	0.07	2.74	0.21	0.174	3.75	7.69
2	40.	44.35	0.10	0.03	1.33	0.11	0.164	3.54	5.76
1	0.	49.49	0.	0.	0.	0.	0.153	3.30	3.91

(1) From Table B.2

(2) From Fig. 3(a) or Table 1(a)

(3) From step 6, by linearly interpolating the data of Fig. 12 or Table 6

(4) gp = 0 at y = 381 ft, the free surface of water, and varies linearly to 1.98 at level 13

(5) From Eq. 1.

Stress Computation -- Fundamental Mode

The equivalent lateral earthquake forces $f_1(y)$ consist of forces associated with the mass of the dam (the first term of Eq. 1) and the hydrodynamic pressure at the upstream face (the second term). For the purpose of computing bending stresses in the monolith, the forces associated with the mass are applied at the centroids of the blocks. The forces due to the hydrodynamic pressure are applied as a linearly distributed load to the upstream face of each block. Due to these two sets of lateral forces, the resultant bending moments in the monolith are computed at each level from the equations of equilibrium. The normal bending stresses are obtained from elementary beam theory. A computer program (described in Appendix C) was developed for computation of the normal bending stresses in a dam monolith due to equivalent lateral earthquake forces. This is a modified version of the computer program prsented in Ref. [2]. However, an alternative approach in which $f_1(y)$ is computed at the top and bottom of each block is more suitable for hand calculation since it avoids computing the location of the centroid of each block. Using this alternative procedure, the forces $f_1(y)$ and the normal bending stresses σ_{y1} at the two faces of Pine Flat Dam associated with the fundamental vibration mode response of the dam to the earthquake ground motion characterized by the smooth design spectrum of Fig. 15 were computed (Tables B.3 and B.4).

Equivalent Lateral Forces -- Higher Vibration Modes

The equivalent lateral earthquake forces $f_{sc}(y)$ due to the higher vibration modes are given by Eq. 9, evaluated at each level using the maximum ground acceleration for the design earthquake, $a_g = 0.25$ g, and $L_1/M_1 = 2.90$ and $B_1/M_1 = 1.837$. The results are summarized in Table B.5. The calculation of bending moments due to the higher vibration modes is similar to the moment calculations for the fundamental vibration mode, as described previously.

Table B.4 -- Normal Bending Stresses
-- Fundamental Vibration Mode

Level	(1) Section Modulus (ft^3)	Bending Moment (k-ft)	Bending Stress at Faces (psi)
10	653.1	12070.	128.
9	823.7	16500.	139.
8	1259.	30040.	166.
7	2404.	72730.	210.
6	3917.	137100.	243.
5	5797.	222400.	266.
4	8045.	326100.	281.
3	10660.	445400.	290.
2	13640.	577000.	294.
1	16990.	717900.	293.

(1) From Table B.2.

Table B.5 -- Equivalent Lateral Earthquake Forces
-- Higher Vibration Modes

Level	y (ft)	(1) $w_s[1-\frac{L_1}{M_1}\phi]$ (k/ft)	y/H	(2) $\frac{gp_o}{wH}$	gp_o (k/ft)	(3) $[gp_o-\frac{B_1}{M_1}w_s\phi]$ (k/ft)	(4) $f_{sc}(y)$ (k/ft)
Top	400.	-1.41	1.05	0.	0.	-1.36	-0.69
14	389.38	-1.30	1.02	0.	0.	-1.29	-0.65
13	359.286	-2.06	0.94	0.15	3.54	1.28	-0.20
12	340.	-1.68	0.89	0.24	5.61	3.59	0.48
11	335.	-5.66	0.88	0.26	6.09	-0.90	-1.64
10	308.85	-7.34	0.81	0.35	8.30	-2.48	-2.46
9	300.	-7.20	0.79	0.38	8.96	-2.49	-2.42
8	280.	-6.24	0.74	0.43	10.27	-2.20	-2.11
7	240.	-1.98	0.63	0.52	12.46	-0.57	-0.64
6	200.	4.64	0.53	0.60	14.15	2.08	1.68
5	160.	12.79	0.42	0.65	15.45	5.26	4.51
4	120.	21.89	0.32	0.69	16.43	8.74	7.66
3	80.	31.23	0.21	0.72	17.12	12.08	10.83
2	40.	40.49	0.11	0.74	17.50	15.05	13.89
1	0.	49.49	0.	0.74	17.64	17.64	16.78

(1) w_s and ϕ from Table B.3

(2) From linear interpolation of data from Fig. 13 or Table 8

(3) $w_s\phi$ from Table B.3; $gp_o = 0$ at $y = 381$ ft, the free surface of water, and varies linearly to 3.54 at level 13

(4) From Eq. 9.

Stress Computation -- Higher Modes

The normal bending stresses at the faces of the monolith due to the equivalent lateral earthquake forces $f_{sc}(y)$ are computed by the procedure described above for stresses due to forces $f_1(y)$. The resulting normal bending stresses $\sigma_{y,sc}$ presented in Table B.6 are due to the response contributions of the higher vibration modes.

Table B.6 -- Normal Bending Stresses
-- Higher Vibration Modes

Level	Section Modulus (1) (ft^3)	Bending Moment (k-ft)	Bending Stress at Faces (psi)
10	653.1	-2382.	-25.
9	823.7	-3181.	-27.
8	1259.	-5649.	-31.
7	2404.	-12600.	-36.
6	3917.	-20410.	-36.
5	5797.	-25440.	-30.
4	8045.	-23160.	-20.
3	10660.	-8605.	-6.
2	13640.	23290.	12.
1	16990.	77300.	32.

(1) From Table B.2.

APPENDIX C: COMPUTER PROGRAM FOR STRESS COMPUTATION

This appendix describes a computer program for computing the stresses in a nonoverflow or a spillway monolith of a concrete gravity dam using the results of the step-by-step simplified analysis procedure presented in this report. The program computes the bending stresses due to the equivalent lateral forces, $f_1(y)$ and $f_{sc}(y)$, representing the maximum effects of the fundamental and higher vibration modes of the dam, respectively. The program also computes the direct and bending stresses due to the self-weight of the dam and hydrostatic pressure. Transformation to principal stresses and combination of stresses due to the three load cases are not performed.

The program is written in FORTRAN 77 for interactive execution.

Simplified Model of Dam Monolith

A dam monolith is modeled as a series of blocks, numbered sequentially from the base to the crest. Increasing the number of blocks increases the accuracy of the computed stresses. The free surface of the impounded water may be at any elevation. The elevation of the reservoir bottom must be equal to the elevation of a block bottom. Fig. C.1 shows the features of the simplified block model.

Program Input

The program queries the user for all input data, which are entered free-format. The program assumes that the unit of length is feet, the unit of force and weight is kips, the unit of acceleration is g's, and the unit of stress is psi. The input data are as follows:

1. ITYPE, to identify the type of dam monolith. ITYPE = 1 for non-overflow monolith; ITYPE = 2 for spillway monolith.
2. N, the total number of blocks in the simplified model.

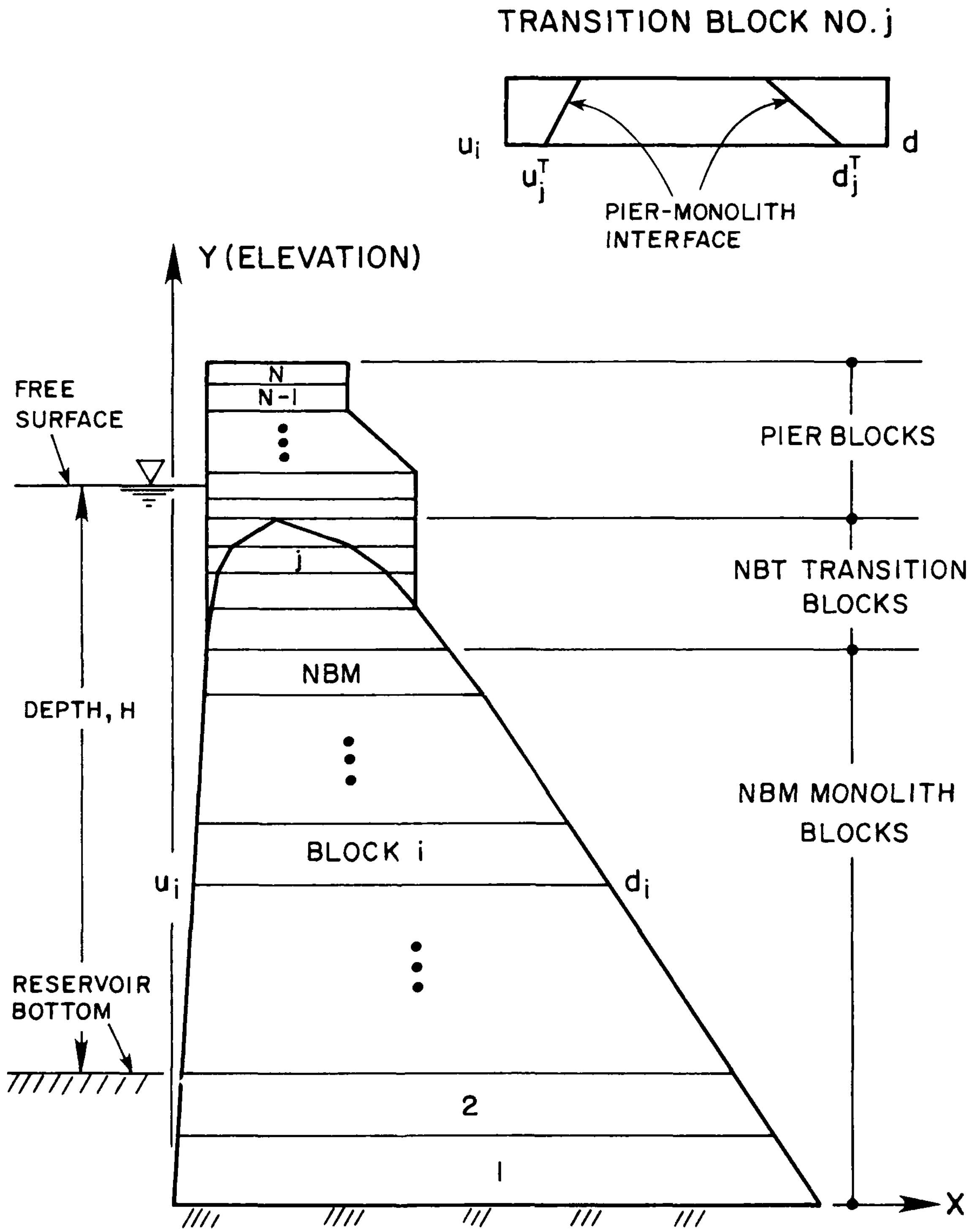

Figure C1 -- Block Model of Spillway Section

3. NBM, the number of blocks in the monolith portion of spillway section. This input is queried only for ITYPE = 2.
4. NBT, the number of transition blocks containing parts from both monolith and pier of the spillway section. This input is queried only for ITYPE = 2.
5. The ratio of the width of pier to the width of monolith, queried only for ITYPE = 2.
6. The default unit weight of concrete in the dam.
7. For the bottom of each block i, the x-coordinate u_i at the upstream face, the x-coordinate d_i at the downstream face, the elevation, and the unit weight of concrete in the block (enter zero if default unit weight).
8. For block j = NBM+1 to NBM+NBT+1, the x-coordinates u_j^T and d_j^T of the upstream and downstream interfaces of the monolith and the pier of the spillway section, respectively, queried only for ITYPE = 2.
9. The x-coordinates of the upstream and downstream faces and the elevation of the dam crest.
10. An alternate value for the ratio L_1/M_1, if desired, where M_1 and L_1 are the generalized mass and earthquake force coefficient for the dam on rigid foundation rock with empty reservoir (Eqs. 2b and 3b). If not specified, the value of L_1/M_1 computed from the block model (as in Steps 7 and 8 of the step-by-step procedure) is used.

The remaining data are entered for each case:

11. The elevations of the free surface of water and reservoir bottom.
12. The ordinates of the hydrodynamic pressure function, gp/wH, at the y/H values indicated. The ordinates are obtained from Step 6 of the step-by-step procedure.
13. The pseudo-acceleration ordinate of the earthquake design spectrum evaluated at the fundamental vibration period and damping ratio of the dam as evaluated in Step 9 of the step-by-step procedure.

14. The ratio $\tilde{L}_1/\tilde{M}_1$, where $\tilde{M}_1$ and $\tilde{L}_1$ = the generalized mass and earthquake force coefficient, including hydrodynamic effects determined in Steps 7 and 8 of the step-by-step procedure. This ratio reduces to L_1/M_1 for a dam with empty reservoir.

15. An alternate value of B_1/M_1, if desired. If not specified, the value computed in Step 11 of the step-by-step procedure is used.

16. The maximum ground acceleration of the design earthquake.

Computed Response

The program computes the vertical, normal (bending) stresses at the bottom of each block of the monolith at the upstream and downstream faces based on simple beam theory. The stresses are not computed in the pier portion of an overflow section. Stresses are computed for three loading cases: (1) static forces (self-weight of the dam and hydrostatic pressure); (2) equivalent lateral forces associated with the fundamental vibration mode; and (3) the equivalent lateral forces associated with the higher vibration modes. The unit of stress is pounds per square inch.

Example

The use of the computer program in the stress computation for the spillway section of Pine Flat Dam is illustrated in the listing shown next wherein the computed vertical, normal (bending) stresses due to the four loading cases are also presented.

```
ENTER DAM TYPE
( 1 = NON-OVERFLOW SECTION, 2 = OVERFLOW SECTION ):2

ENTER THE NUMBER OF BLOCKS IN THE DAM: 14

     ENTER NO. OF BLOCKS OF MONOLITH: 9

     ENTER NO. OF TRANSITION BLOCKS: 2

     ENTER WIDTH RATIO OF PIER AND MONOLITH: .16

ENTER THE DEFAULT UNIT WEIGHT: .155

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  1: 0,319.283,0,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  2: 2,288.082,40,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  3: 4,256.881,80,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  4: 6,225.679,120,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  5: 8,194.478,160,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  6: 10,163.277,200,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  7: 12,132.076,240,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  8: 14,100.874,280,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.  9: 15,85.274,300,0

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO. 10: 15.4425,78,308.85,0

     ENTER X1 AND X2 OF TRANSITION LEVEL: 15.4425,78

     ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO. 11: 16.75,73.335,0

     ENTER X1 AND X2 OF TRANSITION LEVEL: 16.75,46.434
```

ENTER X1 AND X2 OF TRANSITION LEVEL: 27.75,27.75

ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO. 13: 16.75,78,359.286,0

ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO. 14: 16.75,46.75,389.380,0

ENTER X1,X2, AND Y AT THE CREST: 16.75,46.75,400

CHECK INPUT DATA :
NBLOCK = 14 NBM = 9 NBT = 2

BLOCK	XLEFT	XTRAN	XTRAN	XRIGHT	Y
	16.750			46.750	400.000
14					
	16.750	16.750	46.750	46.750	389.380
13					
	16.750	16.750	78.000	78.000	359.286
12					
	16.750	27.750	27.750	78.000	340.000
11					
	16.750	16.750	46.434	78.000	335.000
10					
	15.443	15.443	78.000	78.000	308.850
9					
	15.000	15.000	85.274	85.274	300.000
8					
	14.000	14.000	100.874	100.874	280.000
7					
	12.000	12.000	132.076	132.076	240.000
6					
	10.000	10.000	163.277	163.277	200.000
5					
	8.000	8.000	194.478	194.478	160.000
4					
	6.000	6.000	225.679	225.679	120.000
3					
	4.000	4.000	256.881	256.881	80.000
2					
	2.000	2.000	288.082	288.082	40.000
1					
	0.000	0.000	319.283	319.283	0.000

PROPERTIES OF THE DAM

LOCK	CENTROID ELEV.	WEIGHT
14	394.690	7.901
13	372.615	34.051
12	349.643	29.295
11	337.033	17.257
10	320.679	197.174
9	304.339	91.106
8	289.648	243.579
7	258.930	641.545

6	219.190	847.394
5	179.349	1053.240
4	139.455	1259.087
3	99.532	1464.936
2	59.589	1670.785
1	19.634	1876.631

		9433.984

FUNDAMENTAL VIBRATION PROPERTIES OF THE DAM

L1 = 1623.176   M1 = 559.265

THE FACTOR L1/M1 = 2.902

ENTER AN ALTERNATE VALUE FOR L1/M1:0

DO YOU WANT TO CONTINUE? (0=YES,1=NO):0

ENTER ELEVATION OF FREE-SURFACE: 0

ENTER ELEVATION OF RESERVOIR BOTTOM:0

STATIC STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-47.681	-15.745
9	-59.408	-15.059
8	-84.021	-15.159
7	-129.991	-15.971
6	-173.756	-17.375
5	-216.414	-19.133
4	-258.440	-21.111
3	-300.073	-23.232
2	-341.444	-25.455
1	-382.632	-27.749

ENTER THE PSUEDO-ACCELERATION ORDINATE IN G: .677

ENTER L1(TILDE)/M1(TILDE) FACTOR: 2.90

FUNDAMENTAL MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	111.618	-111.618
9	121.655	-121.655
8	148.300	-148.300

7	195.001	-195.001
6	230.668	-230.668
5	255.373	-255.373
4	270.639	-270.639
3	278.129	-278.129
2	[illegible]	[illegible]
1	275.944	-275.944

THE FACTOR B1/M1 IS =   0.000
ENTER AN ALTERNATE VALUE FOR B1/M1:0

ENTER MAX. GROUND ACCELERATION IN G: .25

HIGHER MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-22.983	22.983
9	-24.317	24.317
8	-27.655	27.655
7	-31.153	31.153
6	-30.549	30.549
5	-26.045	26.045
4	-18.189	18.189
3	-7.564	7.564
2	5.276	-5.276
1	19.813	-19.813

DO YOU WANT TO CONTINUE? (0=YES,1=NO):0

ENTER ELEVATION OF FREE-SURFACE: 381

ENTER ELEVATION OF RESERVOIR BOTTOM:0

STATIC STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-8.202	-56.294
9	-15.470	-60.368
8	-28.813	-72.384
7	-51.994	-97.173
6	-72.818	-122.644
5	-92.466	-148.509
4	-111.445	-174.616
3	-130.008	-200.879
2	-148.294	-227.252
1	-166.386	-253.702

ENTER THE HYDRODYNAMIC PRESSURE FOR THE
FUNDAMENTAL VIBRATION MODE OF THE DAM

ENTER THE PRESSURE ORDINATE FOR Y/H =0.943: .092

ENTER THE PRESSURE ORDINATE FOR Y/H =0.892: .144

ENTER THE PRESSURE ORDINATE FOR Y/H =0.879: .151

ENTER THE PRESSURE ORDINATE FOR Y/H =0.811: .178

ENTER THE PRESSURE ORDINATE FOR Y/H =0.787: .185

ENTER THE PRESSURE ORDINATE FOR Y/H =0.735: .199

ENTER THE PRESSURE ORDINATE FOR Y/H =0.630: .209

ENTER THE PRESSURE ORDINATE FOR Y/H =0.525: .209

ENTER THE PRESSURE ORDINATE FOR Y/H =0.420: .198

ENTER THE PRESSURE ORDINATE FOR Y/H =0.315: .189

ENTER THE PRESSURE ORDINATE FOR Y/H =0.210: .174

ENTER THE PRESSURE ORDINATE FOR Y/H =0.105: .164

ENTER THE PRESSURE ORDINATE FOR Y/H =0.000: .153

ENTER THE PSUEDO-ACCELERATION ORDINATE IN G: .542

ENTER L1(TILDE)/M1(TILDE) FACTOR: 3.14

FUNDAMENTAL MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	184.558	-184.558
9	200.011	-200.011
8	238.321	-238.321
7	302.053	-302.053
6	349.554	-349.554
5	382.941	-382.941
4	404.679	-404.679
3	417.126	-417.126
2	422.237	-422.237
1	421.778	-421.778

THE FACTOR B1/M1 IS = 1.837
ENTER AN ALTERNATE VALUE FOR B1/M1:0

ENTER MAX. GROUND ACCELERATION IN G: .25

HIGHER MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-25.332	25.332
9	-26.820	26.820
8	-31.167	31.167
7	-36.386	36.386
6	-36.179	36.179
5	-30.472	30.472
4	-19.995	19.995
3	-5.606	5.606
2	11.857	-11.857
1	31.593	-31.593

DO YOU WANT TO CONTINUE? (0=YES,1=NO):0

ENTER ELEVATION OF FREE-SURFACE: 0

ENTER ELEVATION OF RESERVOIR BOTTOM:0

STATIC STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-47.681	-15.745
9	-59.408	-15.059
8	-84.021	-15.159
7	-129.991	-15.971
6	-173.756	-17.375
5	-216.414	-19.133
4	-258.440	-21.111
3	-300.073	-23.232
2	-341.444	-25.455
1	-382.632	-27.749

ENTER THE PSUEDO-ACCELERATION ORDINATE IN G: .453

ENTER L1(TILDE)/M1(TILDE) FACTOR: 2.90

FUNDAMENTAL MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	74.686	-74.686
9	81.403	-81.403
8	99.232	-99.232
7	130.481	-130.481
6	154.346	-154.346
5	170.877	-170.877
4	181.092	-181.092
3	186.104	-186.104
2	186.963	-186.963
1	184.642	-184.642

THE FACTOR B1/M1 IS = 0.000
ENTER AN ALTERNATE VALUE FOR B1/M1:0

ENTER MAX. GROUND ACCELERATION IN G: .25

HIGHER MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-22.983	22.983
9	-24.317	24.317
8	-27.655	27.655
7	-31.153	31.153
6	-30.549	30.549
5	-26.045	26.045
4	-18.189	18.189
3	-7.564	7.564
2	5.276	-5.276
1	19.813	-19.813

DO YOU WANT TO CONTINUE? (0=YES,1=NO):0

ENTER ELEVATION OF FREE-SURFACE: 381

ENTER ELEVATION OF RESERVOIR BOTTOM:0

STATIC STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-8.202	-56.294
9	-15.470	-60.368
8	-28.813	-72.384

7	-51.994	-97.173
6	-72.818	-122.644
5	-92.466	-148.509
4	-111.445	-174.616
3	-130.008	-200.879
2	-148.294	-227.252
1	-166.386	-253.702

ENTER THE HYDRODYNAMIC PRESSURE FOR THE
FUNDAMENTAL VIBRATION MODE OF THE DAM

ENTER THE PRESSURE ORDINATE FOR Y/H =0.943: .092

ENTER THE PRESSURE ORDINATE FOR Y/H =0.892: .144

ENTER THE PRESSURE ORDINATE FOR Y/H =0.879: .151

ENTER THE PRESSURE ORDINATE FOR Y/H =0.811: .178

ENTER THE PRESSURE ORDINATE FOR Y/H =0.787: .185

ENTER THE PRESSURE ORDINATE FOR Y/H =0.735: .199

ENTER THE PRESSURE ORDINATE FOR Y/H =0.630: .209

ENTER THE PRESSURE ORDINATE FOR Y/H =0.525: .209

ENTER THE PRESSURE ORDINATE FOR Y/H =0.420: .198

ENTER THE PRESSURE ORDINATE FOR Y/H =0.315: .189

ENTER THE PRESSURE ORDINATE FOR Y/H =0.210: .174

ENTER THE PRESSURE ORDINATE FOR Y/H =0.105: .164

ENTER THE PRESSURE ORDINATE FOR Y/H =0.000: .153

ENTER THE PSUEDO-ACCELERATION ORDINATE IN G: .377

ENTER L1(TILDE)/M1(TILDE) FACTOR: 3.14

FUNDAMENTAL MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	128.373	-128.373

9	139.122	-139.122
8	165.769	-165.769
7	210.099	-210.099
6	243.140	-243.140
5	266.363	-266.363
4	281.483	-281.483
3	290.141	-290.141
2	293.696	-293.696
1	293.377	-293.377

THE FACTOR B1/M1 IS = 1.837
ENTER AN ALTERNATE VALUE FOR B1/M1:0

ENTER MAX. GROUND ACCELERATION IN G: .25

HIGHER MODE STRESSES IN DAM

BLOCK	UPSTREAM FACE	DOWNSTREAM FACE
10	-25.332	25.332
9	-26.820	26.820
8	-31.167	31.167
7	-36.386	36.386
6	-36.179	36.179
5	-30.472	30.472
4	-19.995	19.995
3	-5.606	5.606
2	11.857	-11.857
1	31.593	-31.593

DO YOU WANT TO CONTINUE? (0=YES,1=NO):1

```
C*********************************************************************
C
C         A COMPUTER PROGRAM TO PERFORM A SIMPLIFIED ANALYSIS
C            OF CONCRETE GRAVITY DAMS DUE TO EARTHQUAKES
C          INCLUDING THE EFFECTS OF DAM-WATER INTERACTION,
C   DAM-FOUNDATION ROCK INTERACTION, AND RESERVOIR BOTTOM ABSORPTION
C
C                            HANCHEN TAN
C             THE UNIVERSITY OF CALIFIRNIA AT BERKELEY
C
C                           VERSION 2.0 :
C         A MODIFICATION OF VERSION 1.0, JANUARY 1985 BY
C                          GREGORY FENVES
C
C                           NOVEMBER 1987
C
C*********************************************************************
C
      CALL SIMPL
      STOP
      END
C--------------------------------------------------------------------
      SUBROUTINE SIMPL
C
C         MAIN SUBPROGRAM -- CONTROL THE EXECUTION OF THE PROGRAM
C
      DIMENSION BLOCKS(5,21),PRESS(21),WEIGHT(20),STRSTA(2,20),
     1          STRWGT(2,20),STRDUM(2,20),STRFUN(2,20),
     2          STRDYN(2,20),STRCOR(2,20),STRWCR(2,20)
C
      DIMENSION PARTFC(3)
      CHARACTER*40 TITSTA,TITFUN,TITCOR
C
      DATA TITSTA/'            STATIC STRESSES IN DAM        '/,
     1     TITFUN/'     FUNDAMENTAL MODE STRESSES IN DAM     '/,
     2     TITCOR/'        HIGHER MODE STRESSES IN DAM       '/
C
C         THESE DATA STATEMENTS, AND THE FIRST DIMENSION STATEMENT,
C         DETERMINE THE MAXIMUM NUMBER OF BLOCKS THAT MAY BE USED
C
      DATA NMAX/20/
C
C         THIS DATA STATEMENT CONTAINS UNIT-DEPENDENT CONSTANTS
C
      DATA GAMMA/0.0624/,STRCON/0.144/
C
C         READ THE PROPERTIES OF THE BLOCKS, COMPUTE OTHER BLOCK
C         PROPERTIES, AND COMPUTE THE STATIC STRESSES DUE TO THE
C         WEIGHT AND EFFECTIVE EARTHQUAKE FORCE
C
      CALL DAMPRP (BLOCKS,NBLOCK,NBM,NBT,ITYPE,NMAX,WEIGHT,STRWGT,
     1             STRWCR,STRFUN,PARTFC)
C
C         TEST TO CONTINUE WITH EXECUTION OF PROGRAM
C
```

```
   10 WRITE (*,99)
      READ (*,*) I
      IF (I.NE.0) GO TO 20
C
C          READ IN PROPERTIES OF THE IMPOUNDED WATER
C          AND COMPUTE STRESSES DUE TO HYDROSTATIC PRESSURE
C
      CALL REDWAT (H,HB)
      CALL DAMSTA (BLOCKS,NBLOCK,GAMMA,H,HB,PRESS,STRDUM)
      STRFAC = 1.0/STRCON
      NB = NBLOCK
      IF (ITYPE.EQ.2) NB = NBM + 1
      CALL COMSTA (NB,STRWGT,STRDUM,STRSTA,STRFAC,TITSTA)
C
C          COMPUTE THE DYNAMIC STRESSES DUE TO THE FUNDAMENTAL
C          MODE SHAPE
C
      CALL DAMDYN (BLOCKS,NBLOCK,GAMMA,H,HB,PRESS,STRDUM)
      WRITE (*,97)
      READ (*,*) SA
      STRFAC = SA/STRCON
      WRITE (*,95)
      READ (*,*) SA
      STRFAC = STRFAC*SA
      CALL COMSTA (NB,STRFUN,STRDUM,STRDYN,STRFAC,TITFUN)
C
C          COMPUTE THE HIGHER MODE STRESSES
C
      CALL DAMCOR (BLOCKS,NBLOCK,ITYPE,GAMMA,H,HB,PARTFC(2),STRFUN,
     1             PRESS,STRDUM)
      WRITE (*,94)
      READ (*,*) SA
      STRFAC = SA/STRCON
      CALL COMSTA (NB,STRWCR,STRDUM,STRCOR,STRFAC,TITCOR)
      GO TO 10
C
   20 RETURN
C
   99 FORMAT (/////' DO YOU WANT TO CONTINUE? (0=YES,1=NO):')
   97 FORMAT (//' ENTER THE PSUEDO-ACCELERATION ORDINATE IN G: ')
   95 FORMAT (//' ENTER L1(TILDE)/M1(TILDE) FACTOR: ')
   94 FORMAT (//' ENTER MAX. GROUND ACCELERATION IN G: ')
C
      END
C-----------------------------------------------------------------------
      SUBROUTINE DAMPRP (BLOCKS,NBLOCK,NBM,NBT,ITYPE,NMAX,WEIGHT,
     1                   STRWGT,STRWCR,STRFUN,PARTFC)
C
C          INPUT THE PROPERTIES OF THE DAM, AND COMPUTE THE STATIC
C          AND FUNDAMENTAL VIBRATION PROPERTIES OF THE DAM
C
      DIMENSION BLOCKS(5,1),WEIGHT(1),STRWGT(2,1),
     1          STRWCR(2,1),STRFUN(2,1),PARTFC(3)
      DIMENSION TRANS(2,21)
      EXTERNAL VALWGT,VALHOR
```

```
C
C          INPUT BLOCK PROPERTIES AND COMPUTE STATIC STRESSES
C
      WRITE (*,100)
      READ  (*,*) ITYPE
      IF ( ITYPE.NE.1 .AND. ITYPE.NE.2 ) STOP
C
      CALL REDBLK (BLOCKS,TRANS,NBLOCK,NBM,NBT,ITYPE,NMAX,
     1             WIDTHR,WEIGHT)
      CALL BLCKVL (BLOCKS,TRANS,NBLOCK,NBM,NBT,ITYPE,WIDTHR,
     1             WEIGHT,WEIGHT)
      CALL STRLOD (BLOCKS,NBLOCK,WEIGHT,VALWGT,STRWGT)
      CALL STRLOD (BLOCKS,NBLOCK,WEIGHT,VALHOR,STRWCR)
C
      WRITE (*,99)
      DO 10 J = 1,NBLOCK
          I = NBLOCK + 1 - J
          WRITE (*,98) I, BLOCKS(5,I), WEIGHT(I)
   10     CONTINUE
C
C          COMPUTE THE FUNDAMENTAL VIBRATION PROPERTIES AND STRESSES
C          DUE TO THE EFFECTIVE EARTHQUAKE FORCE
C
      CALL FUNMOD (BLOCKS,NBLOCK,ITYPE,WEIGHT,WEIGHT,XTOT,PARTFC(1),
     1             PARTFC(2))
      CALL STRLOD (BLOCKS,NBLOCK,WEIGHT,VALHOR,STRFUN)
C
      PARTFC(3) = PARTFC(1)/PARTFC(2)
      WRITE (*,97) XTOT,(PARTFC(I),I=1,3)
      READ (*,*) DUM
      IF (DUM.GT.0.0) PARTFC(3) = DUM
C
C          COMPUTE THE HIGHER MODE STRESSES DUE TO THE
C          WEIGHT OF THE DAM
C
      DO 20 J = 1,NBLOCK
          STRWCR(1,J) = STRWCR(1,J) - PARTFC(3)*STRFUN(1,J)
          STRWCR(2,J) = STRWCR(2,J) - PARTFC(3)*STRFUN(2,J)
   20     CONTINUE
C
      RETURN
C
  100 FORMAT (//3X,'ENTER DAM TYPE'//3X,'( 1 = NON-OVERFLOW SECTION,'
     1         ,' 2 = OVERFLOW SECTION ):')
   99 FORMAT (//3X,'PROPERTIES OF THE DAM'//'BLOCK',2X,'CENTROID',
     1         4X,'WEIGHT'/10X,'ELEV.'/1X,27('-')/)
   98 FORMAT (3X,I2,3X,F8.3,3X,F8.3)
   97 FORMAT (19X,'--------'/19X,F8.3//
     1         ' FUNDAMENTAL VIBRATION PROPERTIES OF THE DAM'//
     2         5X,' L1 =',F9.3,5X,' M1 =',F9.3//
     3         5X,' THE FACTOR L1/M1 =',F9.3//
     4         ' ENTER AN ALTERNATE VALUE FOR L1/M1:')
C
      END
C-----------------------------------------------------------------------
```

```
      SUBROUTINE DAMCOR (BLOCKS,NBLOCK,ITYPE,GAMMA,H,HB,XM1,STRFUN,
     1                    PRESS,STRDUM)
C
      DIMENSION BLOCKS(5,1),PRESS(1),STRFUN(2,1),STRDUM(2,1)
      EXTERNAL VALHDY
      DATA BFACT1/0.20/,BFACT2/0.25/
C
      CALL CORPRS (BLOCKS,NBLOCK,H,HB,GAMMA,PRESS)
      CALL STRPRS (BLOCKS,NBLOCK,PRESS,1,H,HB,VALHDY,STRDUM)
C
      D = H - HB
      HS = BLOCKS(3,NBLOCK+1) - BLOCKS(3,1)
      BFACT = BFACT1
      IF (ITYPE.EQ.2) BFACT = BFACT2
      B01M1 = 0.5*BFACT*GAMMA*D*D*D*D/(HS*HS*XM1)
      WRITE (*,99) B01M1
      READ (*,*) D
      IF (D.GT.0.0) B01M1 = D
C
      DO 10 J = 1,NBLOCK
          STRDUM(1,J) = STRDUM(1,J) - B01M1*STRFUN(1,J)
          STRDUM(2,J) = STRDUM(2,J) - B01M1*STRFUN(2,J)
   10     CONTINUE
C
      RETURN
C
   99 FORMAT (//' THE FACTOR B1/M1 IS = ',F9.3/
     1          ' ENTER AN ALTERNATE VALUE FOR B1/M1:')
      END
C-----------------------------------------------------------------------
      SUBROUTINE REDBLK (BLOCKS,TRANS,NBLOCK,NBM,NBT,ITYPE,NMAX,
     1                   WIDTHR,UNITWT)
C
C         READ THE PROPERTIES OF THE BLOCKS IN THE DAM
C
      DIMENSION BLOCKS(5,1),TRANS(2,1),UNITWT(1)
C
      WRITE (*,99)
      READ (*,*) NBLOCK
      IF (NBLOCK.GT.NMAX) GO TO 20
      NBM = NBLOCK
      NBT = 0
      WIDTHR = 1.
      IF (ITYPE.EQ.1) GO TO 5
      WRITE (*,100)
      READ (*,*) NBM
      WRITE (*,101)
      READ (*,*) NBT
          NBMT = NBM + NBT
          WRITE (*,102)
          READ (*,*) WIDTHR
C
    5 WRITE (*,97)
      READ (*,*) DEFWGT
C
```

```
      DO 10 I = 1,NBLOCK
          WRITE (*,95) I
          READ (*,*) (BLOCKS(J,I),J=1,3),UNT
          TRANS(1,I) = BLOCKS(1,I)
          TRANS(2,I) = BLOCKS(2,I)
          IF (I.LE.NBM .OR. I.GT.NBMT+1) GO TO 6
          WRITE (*,103)
          READ (*,*) (TRANS(J,I),J=1,2)
    6     IF (UNT.LE.0.0) UNT = DEFWGT
          UNITWT(I) = UNT
   10     CONTINUE
C
      NBL1 = NBLOCK + 1
      WRITE (*,93)
      READ (*,*) (BLOCKS(J,NBL1),J=1,3)
C
C          CHECK INPUT COORDINATES
C
      WRITE (*,104) NBLOCK,NBM,NBT
      WRITE (*,105) (BLOCKS(J,NBL1),J=1,3)
      DO 30 I = 1,NBLOCK
      N = NBL1 - I
      WRITE (*,106) N
      WRITE (*,107) BLOCKS(1,N),(TRANS(J,N),J=1,2),
     1              (BLOCKS(J,N),J=2,3)
   30 CONTINUE
C
      RETURN
C
C          TOO MANY BLOCKS REQUIRED FOR STORAGE ALLOCATED
C
   20 STOP
C
  107 FORMAT (14X,5F10.3)
  106 FORMAT (6X,I5)
  105 FORMAT (/14X,F10.3,20X,2F10.3)
  104 FORMAT (/5X,' CHECK INPUT DATA :'
     1        /9X,'NBLOCK = ',I3,'  NBM = ',I3,'  NBT = ',I3//
     2        8X,'BLOCK',6X,'XLEFT',5X,'XTRAN',4X,'XTRAN',5X,
     3        'XRIGHT',6X,'Y')
  103 FORMAT (/5X,' ENTER X1 AND X2 OF TRANSITION LEVEL: ')
  102 FORMAT (/5X,' ENTER WIDTH RATIO OF PIER AND MONOLITH: ')
  101 FORMAT (/5X,' ENTER NO. OF TRANSITION BLOCKS: ')
  100 FORMAT (/5X,' ENTER NO. OF BLOCKS OF MONOLITH: ')
   99 FORMAT (/' ENTER THE NUMBER OF BLOCKS IN THE DAM: ')
   97 FORMAT (/' ENTER THE DEFAULT UNIT WEIGHT: ')
   95 FORMAT (/5X,' ENTER X1,X2,Y, AND UNIT WEIGHT OF BLOCK NO.
     1        I2,': ')
   93 FORMAT (//5X,' ENTER X1,X2, AND Y AT THE CREST: ')
C
      END
C-----------------------------------------------------------------------
      SUBROUTINE BLCKVL (BLOCKS,TRANS,NBLOCK,NBM,NBT,ITYPE,WIDTHR,
     1                   UNITWT,WEIGHT)
C
```

```
C         COMPUTE THE LOCATIONS OF THE CENTROIDS AND
C         WEIGHTS OF THE BLOCKS
C
      DIMENSION BLOCKS(5,1),TRANS(2,1),UNITWT(1),WEIGHT(1)
C
C         LOOP OVER THE BLOCKS, ONE AT A TIME, TOP TO BOTTOM
C
      NBMT = NBM + NBT
      DO 10 J = 1,NBLOCK
          I = NBLOCK + 1 - J
          IF (I.LE.NBM .OR. I.GT.NBMT) GO TO 9
          CALL BCETRD (BLOCKS,TRANS,I,UNITWT(I),WIDTHR,
     1                 WEIGHT(I),BLOCKS(4,I),BLOCKS(5,I))
          GO TO 10
C
    9     TOP = BLOCKS(2,I+1) - BLOCKS(1,I+1)
          BOT = BLOCKS(2,I  ) - BLOCKS(1,I  )
          DX  = BLOCKS(1,I+1) - BLOCKS(1,I  )
          DY  = BLOCKS(3,I+1) - BLOCKS(3,I  )
C
          CALL CENTRD (TOP,BOT,0.0,DY,AREA,DUM,DUM,RY)
          BLOCKS(4,I) = BLOCKS(1,I) +
     1                  (2.0*DX*TOP + DX*BOT + TOP*BOT
     2                   + TOP*TOP + BOT*BOT)/
     3                  (3.0*(TOP + BOT))
          BLOCKS(5,I) = BLOCKS(3,I) + RY
C
          WEIGHT(I) = AREA*UNITWT(I)
          IF (I.GT.NBMT) WEIGHT(I) = WEIGHT(I)*WIDTHR
   10     CONTINUE
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE FUNMOD (BLOCKS,NBLOCK,ITYPE,WEIGHT,WPHI,W1,W2,W3)
C
C         COMPUTE THE EFFECTIVE LATERAL LOAD FOR EACH BLOCK
C         AND THE TOTAL WEIGHT, EFFECTIVE EARTHQUAKE FORCE,
C         AND GENERALIZED WEIGHT OF THE DAM
C
      DIMENSION BLOCKS(5,1),WEIGHT(1),WPHI(1)
C
      HS = BLOCKS(3,NBLOCK+1) - BLOCKS(3,1)
      W1 = 0.0
      W2 = 0.0
      W3 = 0.0
C
C         LOOP OVER BLOCKS, ONE AT A TIME, BOTTOM TO TOP
C
      DO 10 I = 1,NBLOCK
          Y = (BLOCKS(5,I) - BLOCKS(3,1))/HS
          CALL PHIONE (Y,PHI,ITYPE)
          W  = WEIGHT(I)
          WP = W*PHI
          WPHI(I) = WP
```

```
          W1  = W1 + W
          W2  = W2 + WP
          W3  = W3 + WP*PHI
   10     CONTINUE
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE PHIONE (Y,PHI,ITYPE)
C
C         OBTAIN THE ORDINATE OF THE FUNDAMENTAL VIBRATION MODE
C         OF THE DAM, USE THE STANDARD MODE SHAPE
C
      DIMENSION PHI1(22),PHI2(22)
      DATA DY/0.05/
      DATA PHI1/0.000 , 0.010 , 0.021 , 0.034 , 0.047 ,
     1          0.065 , 0.084 , 0.108 , 0.135 , 0.165 ,
     2          0.200 , 0.240 , 0.284 , 0.334 , 0.389 ,
     3          0.455 , 0.530 , 0.619 , 0.735 , 0.866 ,
     4          1.000 , 1.000 /
      DATA PHI2/0.000 , 0.016 , 0.030 , 0.048 , 0.070 ,
     1          0.094 , 0.123 , 0.155 , 0.192 , 0.232 ,
     2          0.277 , 0.327 , 0.381 , 0.440 , 0.504 ,
     3          0.572 , 0.646 , 0.725 , 0.816 . 0.909 ,
     4          1.000 , 1.000 /
C
      A   = Y/DY
      I   = IFIX(A) + 1
      A   = FLOAT(I) - A
      IF (ITYPE.EQ.1) GO TO 10
      PHI = A*PHI2(I) + (1.0-A)*PHI2(I+1)
      RETURN
   10 PHI = A*PHI1(I) + (1.0-A)*PHI1(I+1)
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE DAMSTA (BLOCKS,NBLOCK,GAMMA,H,HB,PRESS,STRDUM)
C
C         COMPUTE THE STATIC STRESSES IN THE DAM DUE TO IMPOUNDED WATER
C
      DIMENSION BLOCKS(5,1),PRESS(1),STRDUM(2,1)
      EXTERNAL VALHST
C
C         COMPUTE STATIC STRESSES DUE TO IMPOUNDED WATER
C
      CALL CALHST (BLOCKS,NBLOCK,H,HB,GAMMA,PRESS)
      CALL STRPRS (BLOCKS,NBLOCK,PRESS,1,H,HB,VALHST,STRDUM)
C
C         COMPUTE STATIC STRESSES DUE TO TAILWATER -- NOT
C          IMPLEMENTED IN THIS VERSION OF THE PROGRAM
C
      RETURN
      END
C-----------------------------------------------------------------------
```

```
      SUBROUTINE DAMDYN (BLOCKS,NBLOCK,GAMMA,H,HB,PRESS,STRDUM)
C
C           READ HYDRODYNAMIC PRESSURE AND COMPUTE STRESSES
C           DUE TO THE HYDRODYNAMIC PRESSURE
C
      DIMENSION BLOCKS(5,1),PRESS(1),STRDUM(2,1)
      EXTERNAL VALHDY
C
      CALL REDHDY (BLOCKS,NBLOCK,H,HB,GAMMA,PRESS)
      CALL STRPRS (BLOCKS,NBLOCK,PRESS,1,H,HB,VALHDY,STRDUM)
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE REDWAT (H,HB)
C
C           READ THE ELEVATIONS OF THE RESERVOIR
C
      WRITE (*,99)
      READ (*,*) H
      WRITE (*,98)
      READ (*,*) HB
C
     RETURN
C
   99 FORMAT (//' ENTER ELEVATION OF FREE-SURFACE: ')
   98 FORMAT (/ ' ENTER ELEVATION OF RESERVOIR BOTTOM:')
      END
C-----------------------------------------------------------------------
      SUBROUTINE COMSTA (NBLOCK,STR1,STR2,STR3,STRFAC,TITLE)
C
C           ADD STRESSES STR2 TO STR1 AND PUT IN STR3
C
      DIMENSION STR1(2,1),STR2(2,1),STR3(2,1)
      CHARACTER*40 TITLE
C
      WRITE (*,99) TITLE
C
      DO 10 J = 1,NBLOCK
          I = NBLOCK + 1 - J
          STR3(1,I) = (STR1(1,I) + STR2(1,I))*STRFAC
          STR3(2,I) = (STR1(2,I) + STR2(2,I))*STRFAC
          WRITE (*,98) I,STR3(1,I),STR3(2,I)
C
   10     CONTINUE
C
      RETURN
C
   99 FORMAT (//2X,A40//' BLOCK',5X,'UPSTREAM FACE',2X,'DOWNSTREAM FACE'
     1        /1X,40('-')/)
   98 FORMAT (3X,I2,10X,F8.3,9X,F8.3)
C
      END
C-----------------------------------------------------------------------

      SUBROUTINE CORPRS (BLOCKS,NBLOCK.H,HB,GAMMA,PRESS)
```

```
C
C         COMPUTE THE HYDRODYNAMIC PRESSURE ON THE UPSTREAM FACE OF A
C         RIGID DAM WITH INCOMPRESSIBLE WATER, USED FOR THE
C         COMPUTATION OF HIGHER MODE STRESSES
C
      DIMENSION BLOCKS(5,1),PRESS(1)
C
      DEPTH = H - HB
      IF (DEPTH.LE.0.0) RETURN
      NBL1 = NBLOCK + 1
      HS = BLOCKS(3,NBL1) - BLOCKS(3,1)
C
      DO 10 I = 1,NBL1
          PRESS(I) = 0.0
          Y = (BLOCKS(3,I) - HB)/DEPTH
          IF (Y.GT.1.0.OR.Y.LT.0.0) GO TO 10
          CALL POYFUN (Y,PO)
          PRESS(I) = GAMMA*DEPTH*PO
   10     CONTINUE
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE POYFUN (Y,PO)
C
C         OBTAIN THE HYDRODYNAMIC PRESSURE ON A RIGID DAM WITH
C         INCOMPRESSIBLE WATER
C
      DIMENSION POY(22)
      DATA DY/0.05/,POY/0.742 , 0.741 , 0.737 , 0.731 , 0.722 , 0.711 ,
     1                  0.696 , 0.680 , 0.659 , 0.637 , 0.610 , 0.580 ,
     2                  0.546 , 0.509 , 0.465 , 0.418 , 0.362 , 0.301 ,
     3                  0.224 , 0.137 , 0.000 , 0.000 /
C
      A = Y/DY
      I = IFIX(A) + 1
      A = FLOAT(I) - A
      PO = A*POY(I) + (1.0-A)*POY(I+1)
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE CALHST (BLOCKS,NBLOCK,H,HB,GAMMA,PRESS)
C
C         COMPUTE THE HYDROSTATIC PRESSURE ON THE FACE OF THE DAM
C
      DIMENSION BLOCKS(5,1),PRESS(1)
C
C         LOOP OVER THE BLOCK LEVELS, ONE AT A TIME, BOTTOM TO TOP
C
      NBL1 = NBLOCK + 1
      DO 10 I = 1,NBL1
          PRESS(I) = 0.0
          Y = BLOCKS(3,I)
          IF (Y.LT.H.AND.Y.GE.HB) PRESS(I) = GAMMA*(H-Y)
```

```
   10      CONTINUE
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE REDHDY (BLOCKS,NBLOCK,H,HB,GAMMA,PRESS)
C
C          READ AND COMPUTE THE HYDRODYNAMIC PRESSURE AT THE
C          BLOCK LEVELS ON THE UPSTREAM FACE OF THE DAM
C
      DIMENSION BLOCKS(5,1),PRESS(1)
C
      DEPTH = H - HB
      IF (DEPTH.EQ.0.0) RETURN
C
      NBL1 = NBLOCK + 1
      HHS  = DEPTH/(BLOCKS(3,NBL1) - BLOCKS(3,1))
      HHS2 = HHS*HHS
C
C          LOOP OVER BLOCK LEVELS, ONE AT A TIME, TOP TO BOTTOM
C
      WRITE (*,99)
      DO 10 J = 1,NBL1
          I = NBLOCK + 2 - J
          PRESS(I) = 0.0
          Y = (BLOCKS(3,I) - HB)/DEPTH
          IF (Y.GT.1.0.OR.Y.LT.0.0) GO TO 10
C
C              READ PRESSURE COEFFICIENT AND COMPUTE HYDRODYNAMIC
C              PRESSURE AT THE BLOCK LEVEL
C
          WRITE (*,98) Y
          READ (*,*) P
          PRESS(I) = GAMMA*DEPTH*HHS2*P
C
   10     CONTINUE
C
      RETURN
C
   99 FORMAT (//' ENTER THE HYDRODYNAMIC PRESSURE FOR THE'/
     1           ' FUNDAMENTAL VIBRATION MODE OF THE DAM')
   98 FORMAT (/5X,' ENTER THE PRESSURE ORDINATE FOR Y/H =',
     1           F5.3,': ')
C
      END
C-----------------------------------------------------------------------
      SUBROUTINE STRLOD (BLOCKS,NBLOCK,LOADS,VALUES,STRESS)
C
C          COMPUTE THE NORMAL STRESSES DUE TO LOADS APPLIED AT THE
C          CENTROID OF THE BLOCKS
C
      DIMENSION BLOCKS(5,1),LOADS(1),STRESS(2,1)
      REAL LOADS,M
C
      HSUM  = 0.0
```

```
      HYSUM = 0.0
      VSUM  = 0.0
      VXSUM = 0.0
C
C         LOOP OVER BLOCKS,ONE AT A TIME, TOP TO BOTTOM
C
      DO 10 J = 1,NBLOCK
          I = NBLOCK + 1 - J
C
C             OBTAIN THE LOADS AT THE CENTROID OF BLOCK I
C
          CALL VALUES (I,LOADS,V,H)
          HSUM  = HSUM  + H
          HYSUM = HYSUM + H*BLOCKS(5,I)
          VSUM  = VSUM  + V
          VXSUM = VXSUM + V*BLOCKS(4,I)
C
C             COMPUTE THE BENDING MOMENT AND STRESSES AT THE
C             BOTTOM OF BLOCK I
C
          M = HYSUM - VXSUM - BLOCKS(3,I)*HSUM
     1        + 0.5*(BLOCKS(2,I)+BLOCKS(1,I))*VSUM
C
          T = BLOCKS(2,I) - BLOCKS(1,I)
          M = 6.0*M/(T*T)
          STRESS(1,I) = VSUM/T + M
          STRESS(2,I) = VSUM/T - M
C
   10     CONTINUE
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE STRPRS (BLOCKS,NBLOCK,PRESS,IUPDN,H,HB,VALUES,STRESS)
C
C         COMPUTE THE NORMAL STRESSES DUE TO PRESSURE APPLIED
C         AT THE FACE, UPSTREAM (IUPDN=1) OR DOWNSTREAM (IUPDN=2)
C         OF THE BLOCKS
C
      DIMENSION BLOCKS(5,1),PRESS(1),STRESS(2,1)
      REAL M
      LOGICAL YCOMP
C
      HSUM  = 0.0
      HYSUM = 0.0
      VSUM  = 0.0
      VXSUM = 0.0
C
      YB = BLOCKS(3,NBLOCK+1)
C
C         LOOP OVER BLOCKS, ONE AT A TIME, TOP TO BOTTOM
C
      DO 40 J = 1,NBLOCK
C
          I = NBLOCK + 1 - J
```

```
      YBT = YB
      YB  = BLOCKS(3,I)
C
      IF (YB.GE.H.OR.YBT.LE.HB) GO TO 30
C
C         THE BLOCK TOUCHES WATER, OBTAIN THE WATER PRESSURE
C         AT THE TOP AND BOTTOM OF THE BLOCK
C
      CALL VALUES (I,PRESS,P1,P2,YCOMP)
      DX = 0.0
      IF (YCOMP) DX = BLOCKS(IUPDN,I+1) - BLOCKS(IUPDN,I)
      DY = BLOCKS(3,I+1) - BLOCKS(3,I)
      IF (YBT.LE.H) GO TO 10
C
C         TOP OF WATER IS IN BLOCK, MODIFY TOP PRESSURE POINT
C
      DUM = H - YB
      DX  = DX*DUM/DY
      DY  = DUM
      P1  = 0.0
      GO TO 20
C
C         CHECK THAT BOTTOM OF WATER CORRESPONDS TO A BLOCK
C
   10 IF (YB.LT.HB) WRITE (*,99)
C
C         COMPUTE PRESSURE AND FORCES ACTING ON BLOCK I
C
   20 CALL CENTRD (P1,P2,DX,DY,H1,V,RX,RY)
C
C         COMPUTE THE STRESS RESULTANTS AT THE BOTTOM OF BLOCK
C
      HSUM  = HSUM  + H1
      HYSUM = HYSUM + H1*(YB+RY)
      VSUM  = VSUM  + V
      VXSUM = VXSUM + V*(BLOCKS(IUPDN,I)+RX)
C
C         COMPUTE THE BENDING MOMENTS AND STRESSES AT THE
C         BOTTOM OF BLOCK I
C
   30 M = HYSUM - VXSUM - BLOCKS(3,I)*HSUM
     1         + 0.5*(BLOCKS(2,I)+BLOCKS(1,I))*VSUM
C
      T = BLOCKS(2,I) - BLOCKS(1,I)
      M = 6.0*M/(T*T)
      STRESS(1,I) = VSUM/T + M
      STRESS(2,I) = VSUM/T - M
C
   40 CONTINUE
C
      RETURN
C
   99 FORMAT (//' ERROR IN MODEL - RESERVOIR BOTTOM DOES NOT'/
     1          ' COINCIDE WITH THE BOTTOM OF A BLOCK'/)
C
```

```
      END
C-----------------------------------------------------------------------
      SUBROUTINE VALWGT (I,LOADS,V,H)
C
C           OBTAIN THE WEIGHT OF BLOCK I
C
      DIMENSION LOADS(1)
      REAL LOADS
C
      H = 0.0
      V = - LOADS(I)
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE VALHOR (I,LOADS,V,H)
C
C           OBTAIN THE EFFECTIVE LATERAL FORCE ON BLOCK I
C
      DIMENSION LOADS(1)
      REAL LOADS
C
      H = LOADS(I)
      V = 0.0
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE VALHST (I,PRESS,P1,P2,YCOMP)
C
C           OBTAIN THE HYDROSTATIC PRESSURE ON BLOCK I
C
      DIMENSION PRESS(1)
      LOGICAL YCOMP
C
      P1 = PRESS(I+1)
      P2 = PRESS(I  )
      YCOMP = .TRUE.
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE VALHDY (I,PRESS,P1,P2,YCOMP)
C
C           OBTAIN THE HYDRODYNAMIC PRESSURE ON BLOCK I
C
      DIMENSION PRESS(1)
      LOGICAL YCOMP
C
      P1 = PRESS(I+1)
      P2 = PRESS(I  )
      YCOMP  = .FALSE.
C
      RETURN
      END
```

```
C-----------------------------------------------------------------------
      SUBROUTINE CENTRD (P1,P2,DX,DY,PX,PY,RX,RY)
C
C          COMPUTE THE RESULTANT PRESSURE FORCE ON A SURFACE.
C          ALSO LOCATES THE VERTICAL CENTROID OF A BLOCK
C
      A  = 0.5*(P1+P2)
      PX =   A*DY
      PY = - A*DX
C
      RX = 0.0
      RY = 0.0
      IF (A.EQ.0.0) RETURN
C
      A  = (2.0*P1+P2)/(6.0*A)
      RX = A*DX
      RY = A*DY
C
      RETURN
      END
C-----------------------------------------------------------------------
      SUBROUTINE BCETRD (BLOCKS,TRANS,JB,UW,WRATIO,WT,CX,CY)
C
C          COMPUTE THE WEIGHT AND LOCATE THE WEIGHT CENTROID
C          OF A TRANSITION BLOCK
C
      DIMENSION BLOCKS(5,1),TRANS(2,1)
      UWR = UW*WRATIO
      CALL WCETRD (BLOCKS(1,JB),TRANS(1,JB),BLOCKS(1,JB+1),TRANS(1,JB+1)
     1            ,BLOCKS(3,JB),BLOCKS(3,JB+1),UWR,WT1,C1X,C1Y)
      CALL WCETRD (TRANS(1,JB),TRANS(2,JB),TRANS(1,JB+1),TRANS(2,JB+1)
     1            ,BLOCKS(3,JB),BLOCKS(3,JB+1),UW ,WT2,C2X,C2Y)
      CALL WCETRD (TRANS(2,JB),BLOCKS(2,JB),TRANS(2,JB+1),BLOCKS(2.JB+1)
     1            ,BLOCKS(3,JB),BLOCKS(3,JB+1),UWR,WT3,C3X,C3Y)
      WT = WT1+WT2+WT3
      CX = (C1X*WT1+C2X*WT2+C3X*WT3)/WT - BLOCKS(1,1)
      CY = (C1Y*WT1+C2Y*WT2+C3Y*WT3)/WT - BLOCKS(3,1)
      RETURN
      END
C-----------------------------------------------------------------------

      SUBROUTINE WCETRD (X1,X2,X3,X4,Y1,Y2,UW,WT,CX,CY)
C
C          COMPUTE THE WEIGHT AND THE WEIGHT CENTROID OF
C          A BLOCK OF TRIANGULAR OF TRAPEZOIDAL SHAPE
C
      DX12  = X2-X1
      DX34  = X4-X3
      DY    = Y2-Y1
      C1X   = (X1+X2+X4)/3.
      C1Y   = (Y1+Y1+Y2)/3.
      C2X   = (X1+X3+X4)/3.
      C2Y   = (Y1+Y2+Y2)/3.
      AREA1 = 0.5*DX12*DY
      AREA2 = 0.5*DX34*DY
      WA1   = UW*AREA1
```

```
      WA2    = UW*AREA2
      WT     = WA1+WA2
      IF (WT.NE.0.0) GO TO 10
      CX     = 0.5*(X1+X3)
      CY     = 0.5*(Y1+Y2)
      RETURN
   10 CX     = (C1X*WA1+C2X*WA2)/WT
      CY     = (C1Y*WA1+C2Y*WA2)/WT
      RETURN
      END
```